AN INTRODUCTION

TO

MECHANICAL DRAWING

BY

FREDERIC R. HONEY,

INSTRUCTOR IN DESCRIPTIVE GEOMETRY AND MECHANICAL DRAWING IN THE SHEFFIELD SCIENTIFIC SCHOOL,

AND

INSTRUCTOR IN PERSPECTIVE IN THE YALE SCHOOL OF THE FINE ARTS.

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NOTE.—The following pages are a reprint of a series of articles which has appeared in the *Educator*. The aim of the writer has been to simplify the subject as far as possible, in order to place it within the reach of those who cannot avail themselves of the services of a teacher. This will account for the minuteness of detail which may be observed throughout the lessons. The plates are those which were designed for the *Educator*, their form and dimensions being adapted to the columns of that paper.

F. R. H.

INTRODUCTORY.

A knowledge of Geometrical Drawing enables the student to represent accurately the true form and dimensions of an object, as well as its appearance as seen from any point of view. We may therefore divide the subject into two parts, viz: Projection Drawing, and Linear Perspective. The characteristic difference between Projection Drawing and Perspective is this: by the former we supply the data, by means of which an object may be constructed; while by the latter we simply represent the appearance of the object. An acquaintance with Projection Drawing is therefore essential to the Engineer, the Architect, and the Artisan. It is also desirable that the Artist should understand the principles of Projection, as a basis for the study of Perspective. Before he can accurately delineate an object as it appears, he must learn to represent its true dimensions. From the foregoing remarks, it will be seen that a course of instruction in Geometrical Drawing should be commenced by a series of lessons on Projections. We will first describe the instruments with which the student should be provided, and offer some remarks upon their use.

CHAPTER I.

THE INSTRUMENTS.

The Drawing Board.—*a b c d*—Fig. I.—should be twenty-four inches long by eighteen inches wide, made of well-seasoned pine or other soft wood, about five-eighths of an inch thick. One side should be battened to prevent it from warping, and the four edges should be straight. It is not necessary that the edges should form an absolutely perfect rectangle,

since the stock of the square should be applied to only one edge in producing any given drawing. The surface should be plane, and free from holes. At a later stage in his work the student may, in addition to the above, provide himself with a board large enough for making a drawing covering twice the area, viz: thirty-six inches by twenty-four inches, but for the present the smaller one will be the most convenient size.

The T Square.—*f*.—Fig. 1.—should be made of well seasoned pear or mahogany or other hard wood, thirty-six inches long. It is composed of two parts, viz: the stock *e* and the blade *f*. The best way to construct this instrument is to place the blade upon the stock, to which it should be fastened by screws, rather than to let it into the stock. The object of this arrangement will be apparent on reference to the illustration, in which it will be seen that the upper surface of the stock becomes a continuation of the upper surface of the drawing board, and no obstruction is offered to the triangle *g* which may slide over it. A T square constructed in this way may be very readily taken apart, and the edges of the stock and blade straightened whenever necessary. It has been already remarked that the stock should be applied to only one edge of the drawing board. It should also be noted that only one edge of the blade, viz: *hi* may be used, and in order that it may be apparent which edge is intended for use, the blade is tapered. If the edges appeared parallel, the student might be tempted to use either, and do inaccurate work. It is not necessary that the working edge of the blade *hi* should be exactly at right angles to the edge of the stock. The essential conditions are that the edge of the stock which is applied to drawing board, and the working edge of the blade should be straight.

The Paper, in commencing, may be the cheaper kind. That known as "G. White," which may be cut into pieces twenty two inches by fifteen inches is a convenient size. It is, desirable that the length and breadth of the paper should be nearly equal to the corresponding dimensions of the drawing board, in order that the holes made by the pins, which fasten the

paper to the board may, as far as possible, be confined to one part of its surface. If a small piece of paper be attached to a large board, it is impossible to avoid making pin holes, into any one of which the point of the divider leg would sink, if it should happen that this point must be used as the centre of a circle.

The Pins or Tacks should be about one-half inch diameter. The heads should be thin and beveled, in order that they may offer the least possible obstruction to the passage of the blade of the T square across the paper.

The Pencil should be two or three *H*, or the equivalent *VH*, and should be cut to a fine wedge-shaped point. A pencil cut in this way will retain its point longer and save the draughtsman a great deal of time. The lines should be drawn lightly in order that they may be easily erased.

The Triangles may be made of well seasoned hard wood, or of rubber. The 45° triangle *g*—Fig. I—is a right angled triangle, two of its angles being equal to each other, viz : 45°. The length of the longest side should be from nine to twelve inches. The 60° triangle *n*—Fig. 1—is a right angled triangle, one of its angles being 60° and the other 30°. The length of the longest side should be from twelve to fifteen inches.

The Curved Ruler.—Fig II.—may be made of well seasoned hard wood, or of rubber, from twelve to fifteen inches long. It should combine short curves at the ends, with long curves at the middle part.

The Ellipsograph.—Fig. III.—an instrument to be used in drawing ellipses, is provided with two needles which may be set at any required distance apart. The needle *a* is fixed to the staff *f*, and the needle *b* which is carried by the saddle *c*, is adjustable, being held to its place by the set-screw *d*. The slot in the staff *f* allows the set-screw *d* to be moved to any desired position. The needles are adjusted so that the points *a* and *b* cover the foci of the ellipse, and then the thread *e* is extended by the pencil point to the required distance, and the curve described as illustrated. When one-half of the curve is drawn, the instrument is reversed and the other half constructed.

The Scale should be made of boxwood, twelve inches long, with a beveled edge divided into inches and parts. The beveled edge makes the use of the scale convenient and accurate, bringing the divisions close to the surface of the paper.

The Parallel Ruler.—Fig IV.—is a contrivance originally designed to be used in drawing section lines, but it may be applied to a great variety of work, as will be evident to any draughtsman. The part *a* is an ordinary 45° triangle, having mounted upon it two parallel guides *b* and *b*, between which slides a tongue *c*, tapered at one end. The part *f* is a *straight edge*, on which are mounted two stops *d* and *d'*, which are parallel respectively to the sides of the tapered end of the tongue *c*, when the triangle and straight edge are applied to each other, as shown in the cut. The triangle *a* and the straight edge *f* are attached to each other by the spiral spring *e*, the effect of which is to keep the stop *d* in contact with the corresponding side of the tapered end of the tongue *c*. In order to draw a series of parallel lines at equal distances apart, place the instrument in the position shown in the cut, and rule a line along the edge of the triangle *gh*. Hold the straight edge firmly in its place and slide the triangle along it, until the side of the tapered end of the tongue comes in contact with the corresponding stop *d'*, and then allow the straight edge to be drawn by the spring *e*: the stop *d* is thus brought in contact with the corresponding edge of the tongue. Rule the second line along the edge *gh*, which will evidently be parallel to the first. The operation may be repeated, and any number of parallel lines drawn, and the distance between them will be regulated by moving the tongue as far as may be desired into the space between the stops *d* and *d'*.

The India Ink may be either liquid or in cake. The former is recommended, since it is prepared expressly for the purpose. Before using it, it is desirable to shake the bottle, in order that the heavier and lighter parts may be mixed together. From time to time one or two drops of water should be added to supply the loss by evaporation. If the cake be employed a saucer will be needed, to which a few drops of

water should be applied. The cake should be gently rubbed on the surface, which must be smooth, and a little water added from time to time, until the requisite amount has been prepared and the ink is black enough for use.

The India Rubber should be soft, in order that in erasing a pencil line the surface of the paper may be raised as little as possible.

The Dividers should be provided with a lengthening bar, a pencil holder and a pen. The needle point is recommended.

The Ruling Pen should have one nib on a hinge, in order that it may be easily cleaned after being used. Before supplying the pen with ink, bring the nibs close together by means of the set-screw, and with an ordinary writing pen place between the nibs the necessary quantity of India ink, The charge should be large, in order that the flow of ink from the pen may be steady, and should be renewed before it is exhausted. Care should be taken to confine the ink to the space between the nibs, in order to avoid the possibility of its being transferred to the straight edge or curved ruler in drawing a line.

The instruments which have been described are all that are necessary for the present. When the student has made some progress he may provide himself with a small bow pencil and pen, the spring instruments, a pair of hair dividers, and beam compasses.

We will conclude this chapter with some practical remarks on the adjustment of the instruments.

In attaching the paper *jklm*—Fig. I—to the board, it is necessary that it shall lie smoothly and be firmly secured, in order that after the drawing is commenced there may be no possibility of its shifting. Put one pin *j* in its place, stretch the paper diagonally from this corner and secure the opposite corner *l*, being careful at the same time to apply the stock of the T square to the left side of the board, and see that the edge of the paper *jk* coincides with the working edge of the blade of the T square. Secure the remaining corners *k* and *m*, being careful to stretch the paper, and bring the heads

of the tacks down upon it. It should be remembered that *the stock of the T square should be applied only to the edge of the drawing board on the left hand of the student*, as shown in the cut. Lines parallel to *jk* should be ruled along the edge *hi* in any part of the paper, the stock *e* being held firmly against the board, and lines perpendicular to these should be ruled along a perpendicular edge of either of the triangles, as *n*, the triangle being applied to the blade of the T square, as shown in the illustration. From this it will be clear that it is not necessary that the edges of the board should form a perfect rectangle, nor that the working edge of the blade of the T square should be perpendicular to the working edge of the stock. Attention is specially called to these points, since draughtsmen often spend time unnecessarily in correcting errors of this kind. But since we may decide to use either edge of the board, it is important that they, as well as the working edges of the stock and blade of the T square, shall be straight.. In order to ascertain if the working edge of the blade of the T square be straight, rule a fine line *o p q* along it, then turn it over, that is, bring the upper surface of the blade into coincidence with the paper, passing the edge *o r q* through the points *o* and *q*. If *o r q* does not coincide with *o p q*, it will be apparent that the edge of the blade should be straightened.

In order to ascertain if the edges of the triangle which are supposed to contain the right angle are perpendicular to each other, apply the triangle to the T square, as shown at *g*, then turn it over into the position *g'*, and if the lines ruled along the edge which is supposed to be perpendicular to the edge of the blade of the T square do not coincide, as shown in the illustration, the edges of the triangle are not perpendicular to each other, and the error should be corrected before the instrument is used. After applying this test to the 45° triangle and correcting the error, if any, ascertain if the remaining angles are equal, as follows: Place the triangle in the position *t s u*, and rule a line *st*; then place the triangle in the position *s' t' u'*, and if *s' t'* does not coincide with *t s*, it will be apparent that the angles are unequal, and the error should be

corrected. After applying the first test to the 60° triangle, and correcting the error, if any, ascertain if the remaining angles are respectively 60° and 30°, as follows: Place the triangle in the position n, and rule a line vw; then place the triangle in the position n', and rule a line $w'\ v'$; from v, the point of intersection of these lines, draw a line $v\ x$ along the edge of the T square; with v as a centre, and with any convenient radius describe an arc, cutting the lines $v\ w$, $w'\ v'$ and $v\ x$ respectively in the points w, v' and x. If the angles are correct the point v' will bisect the arc $w\ x$.

CHAPTER II.

PROJECTIONS.

It was stated in the introduction that a knowledge of Geometrical Drawing enables the student to represent accurately the true form and dimensions of an object. This is accomplished by means of its projections, or in ordinary language, by its plan and elevation. In order to show all the dimensions of an object, we give two views of it, or its representation upon two planes. These planes are taken at right angles to each other.

PLATE A.

Fig. I.—$a\ b\ l\ g$ represents a vertical plane, and is called *the vertical plane of projection.* $g\ l\ c\ d$ represents a horizontal plane, and is called *the horizontal plane of projection.* $g\ l$, the the line of intersection of these planes, is called the *ground line.* Let it be required to represent a point P in space. Let fall a perpendicular from P to the horizontal plane. r the foot of this perpendicular is the projection of the point upon the horizontal plane, and is called its *horizontal projection* or *plan.* Also let fall a perpendicular from P to the vertical plane. q the foot of this perpendicular is the projection of the point on the vertical plane, and is called its *vertical projection* or *elevation.* If the plan and elevation of a point be given the point is located in space. For if we erect from q a perpendicular to

the vertical plane, and from r a perpendicular to the horizontal plane, the intersection of these perpendiculars at P will determine the position of the point in space. We may consider the planes $a\ b\ l\ g$ and $g\ l\ c\ d$ to represent a sheet of drawing paper bent at the line $g\ l$, one part of the paper being at right angles to the other. Now since it would be impracticable to use the drawing paper in this way, we conceive, after the projections of the point are determined as described above, that the vertical plane $a\ b\ l\ g$ is revolved around the ground line $g\ l$ as an axis, until it reaches the position $a'\ b'\ l\ g$, when it becomes a continuation of the horizontal plane $g\ l\ c\ d$. In this position both planes may be represented upon the same plane or sheet of paper $a'\ b'\ c\ d$. It will be observed that during the revolution every point in the vertical plane describes a quadrant of a circle. q the elevation of the point describes a quandrant $q\ s$ and the elevation falls at s. If we let fall from q a perpendicular to the horizontal plane it will pierce it in the ground line at t, and if we complete the rectangle $P\ q\ t\ r$ we see that $s\ t = q\ t = P\ r$; that is, *the distance from the elevation of the point s to the ground line is equal to the distance from the point in space P to to the horizontal plane.* Also $r\ t = P\ q$, that is, *the distance from the plan of the point r to the ground line is equal to the distance from the point in space P, to the vertical plane.*

Let it be required to represent a point at a distance of three inches from the vertical plane, and four inches from the horizontal plane. Let $e\ f\ h\ i$—Fig. II.—represent the drawing board and $a'\ b'\ c\ d$ the drawing paper attached to the board, as described in Chap. I. It should be remembered that the edge of the board $e\ i$ should be on the left hand of the student, as is illustrated in Plate I, which is lettered in the same way as the paper in Fig. II, in order that they may be compared. Apply the stock of the T square to the edge of the board $e\ i$, and rule a line $g\ l$ along the edge of the blade. The part of the paper $a'\ b'\ l\ g$ will represent the vertical plane $a\ b\ l\ g$—Fig. I, and the part $g\ l\ c\ d$ will represent the horizontal plane $g\ l\ c\ d$—Fig. I. Mark s the elevation of the point, at a distance from $g\ l$ equal to four inches, the distance from the point to the hori-

zontal plane. Apply one of the perpendicular edges of either triangle to the edge of the blade of the T square, so that the other perpendicular edge will pass through s, and rule a line $s\ t\ r$ along it, perpendicular to the ground line. From t on this line lay off $t\ r$, equal to three inches, the distance from the point to the vertical plane. r is the plan of the point.

PLATE I.

In this and the succeeding plates the dimensions and positions of the figures will be given, which the student should draw according to the directions. The paper should be twenty-two inches long by fifteen inches wide. Commence by drawing the ground line $g\ l$ across the middle of the paper, and it should be remembered that the part $a'\ b'\ l\ g$ represents the vertical plane, and the part $g\ l\ c\ d$ represents the horizontal plane, and that while these two parts are in the same as plane, they are *supposed* to be at right angles to each other shown in Fig. I, Plate A. If this be continually borne in mind, the student will have little difficulty in understanding the constructions which follow.

Prob. I.—*To draw the plan and elevation of a straight line five inches long, which is perpendicular to the horizontal plane.* The plan of a straight line, which is perpendicular to the horizontal plane $P\ r$—Fig. I, Plate A—is a point r. The plan of a line is a line passing through the horizontal projections of all of its points, and in this particular case all the points are evidently projected in the same point r, that is, the plan of the line is a view of it, looking at it in the direction of the vertical arrow. The eye of the observer is supposed to be in a continuation of the line, and it is plain that only a point will be visible. The elevation $q\ t = s\ t$ is equal to the line, and is a view of it looking at it in the direction of the horizontal arrow.

Therefore assume a point e' at a distance of two inches and a half from the left side of the paper, and four inches from the ground line. From e' draw $e'\ e$ perpendicular to the ground line, and lay off upon it from the ground line $e''\ e$,

equal to five inches. e and $e''e$ are the required plan and elevation.

Prob. II.—*To draw the plan and elevation of a square prism, one of whose bases is situated in the horizontal plane. Length of the side of the base=$2\frac{1}{2}''$. Altitude of prism=$5''$.* Assume a point e' at a distance of seven inches from the left side of the paper, and of five and a half inches from the ground line. From e' draw $e'f'$ and $e'h'$, forming with the ground line respectively angles of 30° and 60°. Lines forming these angles with the ground line should be ruled along the edge of the 60° triangle applied to the edge of the blade of the T square, as indicated by the broken lines. Lines drawn in this way will be perpendicular to each other at the point e'. From e' lay off $e'f'$ and $e'h'$ each equal to two inches and a half—the length of the side of the base. From f' draw $f'i'$ parallel to $e'h'$, and from h' draw $h'i'$ parallel to $e'f'$, intersecting $f'i'$ in i'. These parallel lines may be ruled along the edge of the 60° triangle, sliding it along the edge of the blade of the T square into the desired position, as indicated. The square $e'f'i'h'$ is the plan of the prism. Since the altitude of the prism is five inches, each perpendicular edge is five inches long, and the elevation will be determined as follows: From the points e', f', i', and h', respectively, draw $e'e$, $f'f$, $i'i$ and $h'h$ perpendicular to the ground line, and lay off from the ground line $e''e$, $f''f$, $i''i$ and $h''h$, each equal to five inches. Draw a line from h to f parallel to the ground line, which will evidently pass through the points e and i. hf is the elevation of the upper base of the prism, and that part of the ground line which lies between the points h'' and f'' is the elevation of the lower base, which is situated in the horizontal plane. $hff''h''$ is the elevation required. The ground line is the elevation of the horizontal plane, and therefore contains the elevation of every point which is situated in the horizontal plane, and consequently all surfaces which coincide with it. Also the elevations of all horizontal planes, as the upper base of the prism, will be straight lines parallel to the ground line. The vertical edge, whose plan is i', will be invisible in the

elevation $i\ i''$, and will be represented by a broken line. This will be plain if we consider the direction in which the eye of the observer is supposed to be viewing the object. If we refer to Fig. I, Plate A, we see that the horizontal arrow, which indicates the direction in which the object is seen, corresponds with the arrow in Prob. II, and that only those vertical edges whose plans are h', e' and f' will be visible in the elevation.

Prob. III.—*To draw the plan and elevation of an hexagonal pyramid. Diagonal of hexagon*$=3\frac{1}{2}''$. *Altitude of pyramid*$=$ $5''$. Assume a point o' at a distance of nine inches and a half from the right side of the paper, and four inches from the ground line. With o' as a centre, and with a radius equal to one inch and three-quarters, one-half the length of the diagonal, describe a circle $e'\ f'\ h'\ i'\ j'\ k'$. Commencing at any point e' on the circumference, divide it into six equal parts, in the points e', f', h', i', j' and k'. Draw the sides of the base, $e'\ f'$, $f'\ h'$, $h'\ i'$, $i'\ j'$, $j'\ k'$, and $k'\ e'$, each of which will be equal to the radius $o'\ e'$. Also draw the diagonals $e'\ i'$, $f'j'$, and $h'\ k'$. o'—$e'\ f'\ h'\ i'\ j'\ k'$ is the plan of the pyramid. The point o' is the plan of the vertex, and the lines $o'\ e'$, $o'\ f'$, $o'\ h'$, $o'\ i'$, $o'\ j'$ and $o'\ k'$ the plans of the edges connecting the vertex with the angular points of the base. From o' draw $o'\ o$ perpendicular to the ground line. Lay off $m\ o$ equal to five inches, the altitude of the pyramid. From e', f', h', i', j' and k', respectively, draw $e'\ e$, $f'\ f$, $h'\ h$, $i'\ i$, $j'\ j$ and $k'\ k$, perpendicular to the ground line, intersecting it in e, f, h, i, j and k, the elevations of the angular points of the base. Join o with each of these points. $o\ e\ i$ is the elevation of the pyramid. The visible edges are $o\ e$, $o\ f$, $o\ h$, and $o\ i$, and are represented by full lines, and the invisible edges, $o\ j$ and $o\ k$, are represented by broken lines.

Prob. IV.—*To draw the plan and elevation of an hexagonal prism standing upon an octagonal prism. Make the diagonal of the hexagon* $2\frac{1}{2}''$, *that of the octagon* $4''$, *the altitude of the hexagonal prism* $2''$, *and the height of the octagonal prism* $3''$. Assume a point o at a distance of four inches from the right side of the paper, and four inches from the ground line.

With o as a centre, and with a radius equal to $1\frac{1}{4}''$, one half the diagonal of the hexagon, describe a circle and construct a regular hexagon $e' f' h' i' j' k'$ as described for the preceding problem. With o as a centre and with a radius equal to $2''$, one half the diagonal of the octagon, describe a circle. Divide the circumference of this circle into eight equal parts, in the points m', n', p', q', r', s', t', and v'. Six of these points may be determined, by the placing the $45°$ triangle, in the different positions 1, 2 and 3, in contact with one of the straight edges of the $60°$ triangle w, passing the edges of the triangle through the centre of the circle o, and marking the extremities of the diameters, m', n', p', r', s' and t'. A line through o, parallel to the straight edge w, will cut the circumference in the remaining points v' and q'. Join m' n' &c., &c., $e' f' h' i' j' k'$—$m' n' p' q' r' s' t' v'$, is the plan of the prisms. From v', m' &c., &c., draw v' v, m' m, &c., &c., perpendicular to the ground line, and lay off from the ground line v'' v, m'' m &c., &c., each equal to $3''$, the altitude of the octagonal prism. From v draw v q parallel to the ground line. From e', f' &c., draw e' e, f' f &c., perpendicular to the ground line, and from v q lay off e'' e, f'' f &c., each equal to $2''$, the altitude of the hexagonal prism. From e draw e i parallel to the ground line. e i q q'' v'' v is the elevation of the prisms. In this figure omit those lines which are the elevations of the invisible edges.

After making all the constructions in pencil we may ink the lines. The T square may be dispensed with, and a straight edge of either of the triangles may be used in ruling the lines. The ruling pen should be set for a fine line, and should be tested upon a separate piece of paper before applying it to the drawing. Having set the nibs at the desired distance apart, the pen should not be altered until the drawing is complete, in order that all the lines may be of the same thickness. In ruling a line the pen should be held in a position perpendicular to the surface of the paper, and care should be taken to prevent the ink from coming in contact with the straight edge. This will be accomplished by placing the straight edge a very little distance from and parallel to the line. After all the lines are

drawn as indicated in the plate, the pen should be set for a heavy line, and the ground line *g l* should be inked.

PLATE II.

Prob. I.—*To draw the elevation and the plan of a straight line five inches long, perpendicular to the vertical plane, one extremity of the line being at a distance of one inch from the vertical plane.* The elevation of a straight line which is perpendicular to the vertical plane is a point. The line *Pu*—Fig. 1.—Plate A is supposed to be perpendicular to the vertical plane *a b l g*, and every point of the line is evidently projected in the same point *q*, that is, the elevation of *Pu* is *q*. In drawing the elevation of an object the eye of the observer is supposed to be looking at it in the direction of the horizontal arrow, that is, in a direction perpendicular to the vertical plane. The eye of the observer is supposed to be in a continuation of a line *uP*, and it is plain that only a point will be visible. Since the line is perpendicular to the vertical plane it is parallel to the horizontal plane, and is projected upon it in its true length. Thus *rv*, the plan of the line, is equal to *Pu*, the line in space. *Pu* and *r v* are opposite and therefore equal sides of the rectangle *Pu v r*. The plan of the line is a view of it looking at it in the direction of the vertical arrow.

Therefore assume a point *a* at a distance of three inches from the left side of the paper, and of four inches from the ground line. From *a* draw $a\ a''$ perpendicular to the ground line. Lay off from the ground line to a' a distance equal to one inch, the distance from one extremity of the line to the vertical plane. Also lay off $a'\ a''$ equal to five inches, the length of the given line. The point *a* is the elevation, and a' a'' is the plan of the line.

Prob. II.—*To draw the elevation and the plan of a triangular prism whose bases are parallel to the vertical plane, one base being at a distance of one inch from the vertical plane. Length of sides of base*=3″, 3¾″ *and* 4″ *respectively. Length of prism*=5″.

Since the bases of the prism are parallel to the vertical plane, the edges which are perpendicular to the bases are perpendicular to the vertical plane, and will therefore be projected upon it in points as in the preceding problem. Since the bases of the prism are parallel to the vertical plane the sides of the bases are also parallel to the vertical plane, and will therefore be projected upon in their true length. This will be plain if we refer to Fig. 1—Plate A—*Pu* is parallel to the horizontal plane, and is projected upon it in its true length *rv*.

Draw the elevation of the prism as follows: assume a point *a* at a distance of six inches from the left side of the paper and of four inches from the ground line. From *a* draw *a b*, three inches long, forming with the ground line an angle of 30°. On *a b* as a base construct a triangle *a b c*, *a c* being equal to four inches, and *b c* equal to three inches and three-quarters. The point *c* may be found as follows: with *a* as a centre and a radius equal to four inches describe an arc, and with *b* as a centre and a radius equal to three inches and three-quarters describe an arc intersecting the former in *c*. *a b c* is the elevation of the prism. From *a*, *b* and *c*, respectively draw $a\,a''$, $b\,b''$ and $c\,c''$, perpendicular to the ground line. Lay off from the ground line to a' a distance equal to one inch. From a' draw $a'\,c'$ parallel to the ground line, intersecting bb'' in b' and cc'' in c'. The points a', b' and c' are the plans of the extremities of the edges which are perpendicular to the vertical plane and the line $a'\,c'$ is the plan of a base of the prism. From a' lay off $a'\,a''$ equal to five inches, the length of the prism. From a'' draw $a''\,c''$ parallel to the ground line, intersecting $b'b''$ in b'', and $c'c''$ in c''. The lines $a'a''$, $b'b''$ and $c'c''$ are the plans of the edges which are perpendicular to the vertical plane, and the line $a''\,c''$ is the plan of a base; $a'\,a''\,c''\,c'$ is the plan of the prism.

Prob. III.—*To draw the elevation and plan of a hollow right circular cylinder whose bases are parallel to the vertical plane, one base being at a distance of one inch from the vertical plane. Diameter of cylinder*=$3\frac{1}{2}''$; *diameter of hole*=$2\frac{1}{2}''$. *Length of cylinder*=$5''$.

Assume a point o at a distance of nine inches from the right side of the paper, and of four inches from the ground line. With o as a centre and a radius equal to one inch and a quarter, one half the diameter of the hole, describe the circle cd. With the same centre, and a radius equal to one inch and three quarters, one half the diameter of the cylinder, describe the circle $a\ b$.

Since the bases of the cylinder are parallel to the vertical plane, the axis is perpendicular to the vertical plane, and is projected upon it in a point. The point o is the elevation of the axis. The elements of the cylinder being parallel to the axis are perpendicular to the vertical plane, and are therefore projected upon it in points, and since the elements are equidistant from the axis they are projected in the circumference of a circle. The circle $a\ b$ is the elevation of the outer surface of the cylinder, and the circle $c\ d$ is the elevation of the inner surface; the area between the circles represents the material, of which the thickness is equal to $a\ c$. From the extremities of the horizontal diameters of the circles a, c, d and b respectively, draw aa'', cc'', dd'' and bb'' perpendicular to the ground line. On aa'' lay off from the ground line to a' a distance equal to one inch, the distance from one base of the cylinder to the vertical plane. From a' draw $a'b'$ parallel to the ground line, intersecting cc'', dd'' and bb'' respectively in c', d' and b'. $a'b'$ is the plan of one base of the cylinder. From a' lay off $a'a''$, equal to five inches, the length of the cylinder, and from a'' draw $a''\ b''$ parallel to the ground line, intersecting $c'c''$, $d'd''$ and $b'b''$ respectively in c'', d'' and b''. $a''\ b''$ is the plan of one base of the cylinder. $a'b'\ b''\ a''$ is the plan of the cylinder; $a'a''$ and $b'b''$ are the plans of the extreme visible elements of the outer surface; $c'c''$ and $d'd''$ are the extreme elements of the inner surface, and are represented by broken lines because they are invisible.

Prob. IV.—*To draw the plan of a solid, whose elevation is given; one base being at a distance of one inch from the vertical plane. Length of solid*=5″.

Draw the elevation as follows: assume a point *a* at a distance of five inches from the right side of the paper, and of three inches and a half from the ground line. From *a* draw *a b*, three inches long, forming with the ground line an angle of 30°. From *a* and *b* respectively draw *ag* and *bh*, forming with the ground line angles of 60°, that is draw *ag* and *bh* perpendicular to *ab*; and make *ag* equal to one half inch. Complete the rectangle *abhg*. On *ab* lay off *ac* and *bd*, each equal to one inch and a quarter. From *c* draw *ce*, forming an angle of 60° with the ground line, and make *ce* equal to one inch and a quarter; produce *ec* to *k*, making *ik* equal to one inch and a quarter. Also from *d* draw *df*, forming an angle of 60° with the ground line; produce *fd* to *l* and complete the rectangle *eflk*. It will be observed that all the lines in this figure may be ruled along the edge of the 60° triangle applied to the T square, as is illustrated in Prob. II, Plate I. The figure *afhk* is the elevation which is given. The points *a*, *c*, *e*, &c., &c., are the elevations of edges which are perpendicular to the vertical plane.

From *a*, *e*, *f*, *d*, *b* and *h* respectively draw aa'', ee'', &c., &c., perpendicular to the ground line. On aa'' lay off from the ground line to a' a distance equal to one inch. From a' draw $a'h'$ parallel to the ground line, intersecting hh'' in h'. Lay off $a'a''$ equal to five inches, the length of the solid, and from a'' draw $a''h''$ parallel to the ground line, intersecting hh'' in h''. $a'h'h''a''$ is the required plan.

It will be observed that we have represented only those edges which are visible from the point of sight. The eye of the observer is supposed to be viewing the object in the direction of the arrow which corresponds with the vertical arrow in Fig. I, Plate A.

EXERCISES.

The following exercises are given as tests of the student's knowledge of the constructions contained in Plates I and II.

Exercise I.—*Draw the elevation of a square pyramid standing upon a square pedestal. Altitude of pyramid*$=5''$; *thickness of pedestal*$=1''$. Assume the plan as follows: draw $a'b'$, four

inches long, forming with the ground line an angle of 30°. On $a'b'$ construct the square $a'b'c'd'$. This may be done by drawing $a'd'$ and $b'c'$, each forming an angle of 60° with the ground line, and four inches long, and $c'd'$, forming an angle of 30° with the ground line. Draw the lines $e'f'$ and $g'h'$ parallel to $a'b'$, and at distances from it respectively of $\frac{1}{2}''$ and $3\frac{1}{2}''$. Also draw $f'g'$ and $h'e'$ parallel to $b'c'$, and at distances from it of $\frac{1}{2}''$ and $3\frac{1}{2}''$. Draw the diagonals $e'g'$ and $f'h'$. $a'b'c'd'$ is the plan of the pedestal, and o'—$e'f'g'h'$ the plan of the pyramid.

Draw the elevation, representing the visible edges by full lines, and the invisible edges by broken lines, remembering that the direction in which the object is seen is that indicated by the arrow, which corresponds with the horizontal arrow in Fig. I, Plate A. Since the lower base of the pedestal is situated in the horizontal plane, its elevation will be in the ground line; and since the thickness of the pedestal is one inch, the elevation of the upper base will be a straight line parallel to the ground line, one inch from it; this line will also contain the elevation of the base of the pyramid, because the base of the pyramid is in the plane of the upper base of the pedestal, and since the altitude of the pyramid is five inches, the elevation of the vertex will be a point five inches above the elevation of the upper base of the pedestal, or six inches from the ground line.

Exercise II.—*Draw the plan of a solid whose elevation is given.*

Assume the elevation of the solid as follows: with a centre *o* describe circles of 3″, $4\frac{1}{2}''$ and 6″ diameter; within the circle *abcdefgh* inscribe the regular octagon *abcdefgh*; within the circle *ijklmn* inscribe the regular hexagon *ijklmn*, and within the circle *pqr* inscribe the equilateral triangle *pqr*. The octagon is the elevation of a prism whose length is three inches, the hexagon is the elevation of a prism whose length is two inches, and the triangle is the elevation of a prism whose length is one inch. The bases of the prisms are parallel to the vertical plane; one base of the triangular prism is in the plane of a base of the hexagonal prism, and one base of the hexagonal prism is in the plane of a base of the octagonal

prism, that is the prisms together form a solid. The edges which are represented by the points *a*, *b*, *c*, &c., *i*, *j*, *k*, &c., *p*, *q*, *r*, are perpendicular to the vertical plane. Draw the plan and represent the visible lines *only*, remembering that the direction in which the object is seen is that indicated by the arrow, which corresponds with the vertical arrow in Fig. I, Plate A.

PLATE III.

Prob. I.—*To draw the plan and the elevation of a straight line five inches long, which is parallel to the horizontal plane and at a distance of four inches from it, and which forms with the vertical plane an angle of* 60°. Since the line is parallel to the horizontal plane it will be projected upon it in its true length, and the plan will form with the ground line an angle equal to that formed by the line with the vertical plane. And since the line is parallel to the horizontal plane all its points are equidistant from the horizontal plane, and will be projected on the vertical plane at equal distances from the ground line, that is, the elevation will be a straight line parallel to the ground line. Therefore assume a point a' at a distance of two inches from the left side of the paper, and of two inches from the ground line. From a' draw $a'b'$ five inches long, the length of the given line, forming with the ground line an angle of 60°. Draw *ab* parallel to the ground line, at a distance of four inches from it, the distance from the line to the horizontal plane. From a' draw $a'a$, and from b' draw $b'b$ perpendicular to the ground line, intersecting *ab* respectively in *a* and *b*, the elevation of the extremities of the line. $a'b'$ is the plan, and *ab* the elevation of the given line.

Prob. II.—*To draw the plan and the elevation of a square prism, whose bases are perpendicular to the horizontal plane, the edges which are perpendicular to the bases forming with the vertical plane angles of* 60°. *Length of side of base*=2″; *length of prism*=5″. First draw the elevation and plan of the prism when the edges are perpendicular to the vertical plane. Assume a point *a* No. 1 at a distance of eight inches from the left side of the paper, and four inches from the ground line.

From a draw ab, two inches long, the length of the side of the base, forming with the ground line an angle of 30°. On ab construct the square $abcd$. From a,b,c, and d, respectively, draw aa'', bb'', cc'' and dd'' perpendicular to the ground line. Lay off from the ground line to a' a distance equal to one inch and a half. Draw $a'c'$ parallel to the ground line, the plan of one base of the prism. Lay off $a'a''$ equal to five inches, the length of the prism and draw $a''c''$ parallel to the ground line, the plan of the other base; $abcd$ is the elevation, and $a'c'c''a''$ the plan of the prism.

Now draw the plan and the elevation when the edges form with the vertical plane angles of 60°. Assume a point a''' No. 2, at a distance of seven inches from the right side of the paper, and two inches and a half from the ground line; from a''' draw $a'''a''''$ equal to $a'a''$ No. 1, forming with the ground line an angle of 60°, equal to the angle which the edges of the prism form with the vertical plane. From a''' draw $a'''c'''$ equal to $a'c'$, forming an angle of 30° with the ground line, that is, perpendicular to $a'''a''''$. Also lay off $a'''d'''$ equal to $a'd'$ and $a'''b'''$ equal to $a'b'$. From a'''' draw $a''''c''''$ parallel to $a'''c'''$, and from c''', b''' and d''', respectively draw $c'''c''''$, $b'''b''''$, and $d'''d''''$, parallel to $a'''a''''$. $a'''c'''c''''a''''$, is the plan of the prism which is simply a copy of $a'c'c''a''$. The edges which are perpendicular to the bases are parallel to the horizontal plane; their elevations will therefore be parallel to the ground line, and their distances from the ground line will be equal to the distances respectively from a,b,c and d, to the ground line. Therefore from a,b,c and d, respectively draw $aa_{,,}$, $bb_{,,}$, $cc_{,,}$ and $dd_{,}$ parallel to the ground line. From a''' draw $a'''a_{,}$ and from a'''' draw $a''''a_{,,}$ perpendicular to the ground line, intersecting $a_{,}a_{,,}$ in the points $a_{,}$ and $a_{,,}$. From b''' draw $b'''b_{,}$ and from b'''' draw $b''''b_{,,}$ perpendicular to the ground line, intersecting $b_{,}b_{,,}$ in the points $b_{,}$ and $b_{,,}$. From c''' draw $c'''c_{,}$ and from c'''' draw $c''''c_{,,}$ perpendicular to the ground line, intersecting $c_{,}c_{,,}$ in the points $c_{,}$ and $c_{,,}$. From d''' draw $d'''d_{,}$ and from d'''' draw $d''''d_{,,}$ perpendicular to the ground line, intersecting $d_{,}d_{,,}$ in the points $d_{,}$ and $d_{,,}$.

Join $a_{,,}b_{,,}$, $b_{,,}c_{,,}$, $c_{,,}d_{,,}$ and $d_{,,}a_{,,}$. Also join $a_{,}b_{,}$, $b_{,}c_{,}$, $c_{,}d_{,}$ and $d_{,}a_{,}$. $a_{,}b_{,}b_{,,}c_{,,}d_{,,}d_{,}$ is the elevation required. It should be observed that $a_{,,}b_{,,}$, $a_{,}b_{,}$, $d_{,}c_{,}$ and $d_{,,}c_{,,}$ are parallel to each other; also that $a_{,,}d_{,,}$, $b_{,,}c_{,,}$, $a_{,}d_{,}$ and $b_{,}c_{,}$ are parallel to each other. The lines $b_{,}c_{,}$, $c_{,}d_{,}$ and $c_{,}c_{,,}$ are drawn broken, because they are invisible from the point of sight. This will be plain if we consider that the direction in which the object is seen is that indicated by the arrow.

PLATE IV.

Prob. I.—*To draw the elevation and plan of a straight line six inches long, which is parallel to the vertical plane, and at a distance of four inches from it, and which forms with the horizontal plane an angle of* 60°. Since the line is parallel to the vertical plane it will be projected upon it in its true length, and the elevation will form, with the ground line, an angle equal to that formed by the line with the horizontal plane. And since the line is parallel to the vertical plane all its points are equidistant from the vertical plane, and will be projected on the horizontal plane at equal distances from the ground line; that is, the plan will be a straight line parallel to the ground line. Therefore assume a point a in the ground line, at a distance of five inches from the left side of the paper. From a draw ab six inches long, the given length of the line, forming with the ground line an angle of 60°. Draw $a'b'$ parallel to the ground line, at a distance of four inches from it, the distance from the line to the vertical plane. From a draw aa', and from b draw bb' perpendicular to the ground line, intersecting $a'b'$ respectively in a' and b', the plans of the extremities of the line; ab is the elevation and $a'b'$ the plan of the given line.

Prob. II.—*To draw the elevation and the plan of an hexagonal pyramid, whose axis is parallel to the vertical plane, and whose base forms with the horizontal plane an angle of* 30°. *Length of side of hexagon*$=2''$. *Altitude of pyramid*$=6''$. First draw the plan and elevation of the pyramid when the base coincides with the horizontal plane. Assume a point

o' No. 1 at a distance of eleven inches from the left side of the paper, and of four inches from the ground line. With a centre o', and a radius equal to two inches, describe a circle $a'b'c'd'e'f'$, and within it inscribe the regular hexagon $a'b'c'd'e'f'$. Draw the diagonals of the hexagon, viz. $a'd'$, $b'e'$ and $c'f'$. o'-$a'b'c'd'e'f'$ is the plan of the pyramid. From o' draw $o'o$ perpendicular to the ground line, and lay off from the ground line to o, a distance equal to six inches, the altitude of the pyramid. From the points a', b', c', d', e' and f' draw perpendiculars to the ground line, intersecting it in the points a, b, c, d, e and f. Join oa, ob, oc, od, oe and of. oad is the elevation of the pyramid.

Now draw the elevation and the plan when the base forms with the horizontal plane an angle of 30°. Assume a point $a_{,}$ No. 2 in the ground line, at a distance of five inches from the right side of the paper. From $a_{,}$ draw $a_{,}d_{,}$ equal to ad No. 1, forming an angle of 30° with the ground line. Make $a_{,}o_{,,}$ equal to $ao_{,}$. From $o_{,,}$ draw $o_{,,}o_{,,,}$ perpendicular to $a_{,}d_{,}$, that is, draw $o_{,,}o_{,,,}$, forming an angle of 60° with the ground line. Make $o_{,,}o_{,,,}$ equal to $o_{,}o$. Also lay off $a_{,}f_{,}$ equal to af; $a_{,}b_{,}$ equal to ab; $a_{,}e_{,}$ equal to ae, and $a_{,}c_{,}$ equal to ac. Join $o_{,,,}$ with $a_{,},f_{,},b_{,},e_{,},c_{,}$ and $d_{,}o_{,,,}$. $d_{,}a_{,}$, which is a copy of oad, No. 1, is the elevation of the pyramid. In No. 2 we must understand that the pyramid is supposed to be moved into a position in which the plane of the base forms with the horizontal plane an angle of 30°, each point of the solid being at a distance from the vertical plane, equal to the distance from the vertical plane of the corresponding point in No. 1. That is, the plan of the vertex, and of each of the angular points of the base will be found at a distance from the ground line equal to the distance from the ground line respectively of o',a',b',c',d',e' and f' No. 1. Also the plan of the vertex and of each of the angular points of the base will be found in a perpendicular to the ground line drawn from $o_{,,,},a_{,},b_{,},c_{,},d_{,},e_{,}$ and $f_{,}$ No. 2. We may therefore make the following construction: from o' draw $o'o''$ parallel to the ground line. From $o_{,,,}$ draw $o_{,,,}o''$ perpendicular to the ground line, intersecting

the former in o'' the plan of the vertex. From a', b', c', d', e' and f' respectively, draw $a'a'', b'b'', c'c'', d'd'', e'e''$ and $f'f''$ parallel to the ground line. From $a_{\prime}, b_{\prime}, c_{\prime}, d_{\prime}, e_{\prime}$ and $f_{\prime}$ draw perpendiculars to the ground line intersecting $a'a'', b'b'', c'c'', d'd'', e'e''$ and $f'f''$ respectively, in a'', b'', c'', d'', e'' and f''. Join $a''b'', b''c''$, $c''d'', d''e'', e''f'', f''a''$. Also join $o''a'', o''b'', o''c'', o''d'', o''e'', o''f''$. o''-$a''b''c''d''e''f''$ is the plan of the pyramid. In the hexagon $a''b''c''d''e''f''$ the opposite sides should be parallel. The lines $o''b'', o''c'', o''d'', o''e'', o''f'', f''e'', e''d'', d''c'', c''b''$ should be drawn full, and $o''a'', a''b'', a''f''$ should be drawn broken. This will be plain if we consider that the direction in which the object is seen is that indicated by the arrow.

PLATE V.

Prob. I.—*To draw the plan and the elevation of a circle whose plane is parallel to the vertical plane, and at a distance of four inches from it. Diameter of circle*=4″. The elevation of a circle, whose plane is parallel to the vertical plane, is a circle whose diameter is equal to that of the given circle; and the plan is a straight line, whose length is equal to the diameter of the circle parallel to the ground line, at a distance from it equal to the distance from the plane of the circle to the vertical plane. This is illustrated in Prob. III. Plate II. Therefore assume a point o at a distance of three inches from the left side of the paper, and four inches from the ground line. With o as a centre, and a radius equal to two inches the radius of the given circle describe the circle $a\,b\,c\,d$. Draw $a'c'$ parallel to the ground line, four inches from it, the distance from the plane of the circle to the vertical plane. From a and c respectively the extremities of the horizontal diameter, draw $a\,a'$ and $c\,c'$ perpendicular to the ground line, intersecting $a'c'$ in the points a' and c'; $a\,b\,c\,d$ is the elevation and $a'\,c'$ the plan of the circle.

Prob. II.—*To draw the plan and the elevation of a circle whose plane is perpendicular to the horizontal plane, and which forms with the vertical plane an angle of* 45°. *Diameter of circle*=4″. The plan of a circle whose plane is perpendicular

to the horizontal plane will evidently be a straight line. This is illustrated in Prob. III, Plate II; $a'b'$ is the plan of one base of the cylinder, and $a'' b''$ is the plan of the other base. The plane of each of these circles is perpendicular to the horizontal plane, and being parallel to the vertical plane its plan is drawn parallel to the ground line. The plan of a circle whose plane is perpendicular to the horizontal plane will be a straight line, forming with the ground an angle equal to that formed by the plane of the circle with the vertical plane. Therefore assume a point b' at a distance of ten inches from the left side of the paper, and of four inches from the ground line. Through b' draw $a'c'$ forming an angle of 45° with the ground line. Lay off $b'a'$ and $b'c'$, each equal to two inches, the radius of the circle, that is, make $a'c'$ equal to four inches, the diameter, which is the plan of the circle. The elevation will be found as follows: the point b' is the plan of that diameter which is perpendicular to the horizontal plane; its elevation will therefore be a straight line perpendicular to the ground line, equal to the diameter. From b' draw $b'b$ perpendicular to the ground line. Lay off from the ground line to o, the elevation of the centre, a distance equal to four inches. From o lay off $o\,b$ and $o\,d$, each equal to two inches, the radius of the circle, that is, make $b\,d$ equal to the diameter. The line $a'c'$ which is the plan of the circle is also the plan of that diameter which is parallel to the horizontal plane. The elevation of this diameter will be parallel to the ground line. Therefore through o draw $a\,c$ parallel to the ground line. From a' and c' respectively draw $a'a$ and $c'c$ perpendicular to the ground line, intersecting ac in the points a and c, the extremities of the elevation of the horizontal diameter. The elevation of the circle will be an ellipse, of which bd is the major axis and ac the minor axis. With a or c as a centre, and a radius equal to the semi-major axis or the radius of the circle, that is, two inches, describe an arc intersecting the major axis in the points e and f, the foci of the ellipse. Having determined these points we may construct the curve as described in Chap. I, by the aid of the ellipso-

graph. The points e and f in Prob. II, Plate V, correspond with a and b in Fig. III, (Instruments). $a\ b\ c\ d$ is the elevation required.

Prob. III.—*To draw the plan and the elevation of a right circular cylinder, whose axis is parallel to the horizontal plane, and which forms with the vertical plane an angle of* 45°. *Diameter of cylinder*=4″. *Length*=$5\frac{1}{2}$″. Since the axis of the cylinder is parallel to the horizontal plane it will be projected upon it in its true length, and its plan will form with the ground line an angle equal to that formed by the axis with the vertical plane, viz. 45°. Also since the bases are perpendicular to the axis they are perpendicular to the horizontal plane, and will be projected upon it in straight lines, equal to the diameter of the cylinder, and perpendicular to the plan of the axis. Therefore assume a point b' at a distance of six inches from the right side of the paper, and of six inches from the ground line. From b' draw $b'f'$ five and one half inches long, the given length of the cylinder, forming an angle of 45° with the ground line. Through b' and f' respectively draw $a'\ c'$ and $e'\ g'$, perpendicular to $b'\ f'$, that is, forming with the ground line angles of 45°. Lay off $b'a'$, $b'c'$, $f'g'$ and $f'e'$, each equal to two inches, the radius of the cylinder. Join $a'\ e'$ and $c'\ g'$. $a'\ c'\ g'\ e'$ is the plan of the cylinder, which is a rectangle, whose length is equal to that of the cylinder, and whose width is equal to the diameter The elevation will be determined as follows: Draw $oo_{,}$ parallel to the ground line, four inches from it. From b' and f' respectively draw $b'o$ and $f'o_{,}$ perpendicular to the ground line, intersecting $oo_{,}$ in o and $o_{,}$. $oo_{,}$ is the elevation of the axis, and the points o and $o_{,}$ are the elevations of the centres of the bases. The ellipse, which is the elevation of the circumference of each base, will be constructed as in the preceding problem. bd equal to four inches, the diameter of the cylinder, will be the major axis, and ac the minor axis, determined by erecting perpendiculars to the ground line from the points a' and c', intersecting $o\ o_{,}$ produced in a and c. Similarly the major axis $f\ h$ and the minor axis $e\ g$ are determined. The ellipses $a\ b\ c\ d$ and

e f g h are the elevations of the bases. Draw *b f* and *d h* parallel to the ground line. *a b f g h d* is the elevation of the cylinder. The semi-ellipse *f e h* is drawn broken, because it is invisible from the point of sight, the object being seen in the direction of the arrow.

PLATE VI.

Prob. I.—*To draw the plan and the elevation of a circle, whose plane is parallel to the horizontal plane, and at a distance of four inches from it. Diameter of circle*=4″. It is illustrated in Prob. III, Plate II, that the elevation of a circle whose plane is parallel to the vertical plane is a circle, whose diameter is equal to that of the given circle; also that the plan is a straight line, equal to the diameter, parallel to the ground line, at a distance from it equal to the distance from the plane of the circle to the vertical plane. It is therefore evident that the plan of a circle, whose plane is parallel to the horizontal plane, is a circle whose diameter is equal to that of the given circle; also that the elevation is a straight line, equal to the diameter, parallel to the ground line, at a distance from it equal to the distance from the plane of the circle to the horizontal plane. Therefore assume a point *o* at a distance of three inches from the left side of the paper, and of four inches from the ground line. With *o* as a centre, and with a radius equal to two inches, the radius of the given circle, describe the circle *a′b′c′d′*. Draw *ac* parallel to the ground line, four inches from it, equal to the distance from the plane of the circle to the horizontal plane. From *a′* and *c′* respectively, the extremities of the diameter which is parallel to the vertical plane, draw *a′a* and *c′c* perpendicular to the ground line, intersecting *ac* in the points *a* and *c*. *a′b′c′d′* is the plan, and *ac* the elevation of the circle.

Prob. II.—*To draw the elevation and the plan of a circle, whose plane is perpendicular to the vertical plane, and which forms with the horizontal plane an angle of* 45°. *Diameter of circle*=4″. The plan of a circle, whose plane is perpendicular to the horizontal plane, is a straight line equal to the diameter,

forming with the ground line an angle equal to that formed by the plane of the circle with the vertical plane. This is illustrated in Prob. II, Plate V. Similarly the elevation of a circle, whose plane is perpendicular to the vertical plane is a straight line, equal to the diameter, forming with the ground line an angle equal to that formed by the plane of the circle with the horizontal plane. Therefore assume a point *d* at a distance of nine inches from the left side of the paper, and of four inches from the ground line. Through *d* draw *ac*, forming an angle of 45° with the ground line. Lay off *da* and *dc*, each equal to two inches, the radius of the circle, that is, make *ac* equal to four inches, the diameter, which is the elevation of the circle. The plan will be found as follows: The point *d* is the elevation of that diameter which is perpendicular to the vertical plane; its plan will therefore be a straight line, perpendicular to the ground line, equal to the diameter. From *d* draw *dd'* perpendicular to the ground line. Lay off from the ground line to *o*, the plan of the centre, a distance equal to four inches. From *o* lay off *ob'* and *od'*, each equal to two inches, the radius of the circle, that is, make *b'd'* equal to the diameter. The line *ac*, which is the elevation of the circumference of the circle, is also the elevation of that diameter which is parallel to the vertical plane. The plan of this diameter will be parallel to the ground line. Therefore through *o* draw *a'c'* parallel to the ground line. From *a* and *c* respectively draw *aa'* and *cc'*, perpendicular to the ground line, intersecting *a'c'* in the points *a'* and *c'*, the extremities of the plan of the line. The plan of the circle will be an ellipse, of which *b'd'* is the major axis and *a'c'* the minor axis. The foci of the ellipse may now be determined as in Prob. II, Plate V, and the curve constructed as described in Chapter I, by the aid of the ellipsograph.

Prob. III.—*To draw the elevation and the plan of a right circular cone, whose axis is parallel to the vertical plane, and which forms with the horizontal plane an angle of* 45°. *Altitude of cone*=6″. *Diameter of base*=4″. Since the axis of the cone is parallel to the vertical plane it will be projected

upon it in its true length, and its elevation will form with the ground line an angle equal to that formed by the axis with the horizontal plane, viz. 45°. Also since the base is perpendicular to the axis it is perpendicular to the vertical plane, and will be projected upon it in a straight line, perpendicular to the elevation of the axis. Therefore assume a point *a* on the ground line, at a distance of four inches from the right side of the paper. From *a* draw *ac* forming an angle of 45° with the ground line, and make *ac* equal to four inches, the diameter of the base. Lay off a distance *ab* equal to two inches. From *b* draw *bd* forming an angle of 45° with the ground line; that is draw *bd* perpendicular to *ac*. Lay off *ba* equal to six inches, the height of the cone. Join *da* and *dc*. *dac* is the elevation required. The plan will be found as follows: since the axis is parallel to the vertical plane it will be projected upon the horizontal plane in a straight line, parallel to the ground line. Therefore draw *d'o* parallel to the ground line, at a distance from it equal to four inches. From *d* and *b* respectively draw *dd'* and *bb'*, perpendicular to the ground line, intersecting *d'o* in *d'* and *o*. *d'o* is the plan of the axis, and the point *o* is the plan of the centre of the base. The ellipse *a'b'c'e'*, which is the plan of the circumference of the base, will be constructed as in the preceding problem. *b'e'* equal to four inches, the diameter of the base, will be the major axis, and *a'c'* the minor axis, determined by dropping perpendiculars from the points *a* and *c*, intersecting *dc'* in the points *a'* and *c'*. From *d'* draw *d'g'* and *d'f'*, each tangent to the ellipse. The lines *d'g'* and *d'f'* are the extreme elements of the cone in the plan—the object being seen in the direction of the vertical arrow. The part of the ellipse *f'a'g'* will evidently be drawn broken, since it is invisible from the point of sight. In inking the ellipses in this Plate and in Plate V, a large portion of each curve may be drawn correctly enough for all practical purposes by arcs of circles. *Theoretically* this method of drawing an ellipse is incorrect; but since arcs may be constructed which very nearly coincide with the curve for a considerable distance, it is desirable, in order to make all the parts symmet-

rical to draw them in this way. The construction is made as follows:—Let *ab* and *cd* (Fig. I.) be the major and minor axes of an ellipse. We will suppose the curve drawn in pencil as described in Prob. II., Plate V. Draw two lines *ek* and *eg* (Fig. II.) forming with each other any angle, at the point *e*. With *e* as a centre, and a radius equal to the semi-major axis, viz.: *oa* or *ob*, describe the arc *fg* intersecting *ek* and *eg* respectively in the points *f* and *g*. Also with the same centre and a radius equal to the semi-minor axis, viz: *oc* or *od*, describe the arc *hi* intersecting *ek* and *eg* respectively in the points *h* and *i*. Join *if*, and through *h* and *g* respectively draw *hj* and *gk* parallel to *fi*, intersecting *eg* in *j* and *ek* in *k*. With *a* and *b* the vertices of the major axis as centres, and a radius equal to *ej* describe arcs, intersecting *ab* in *n* and *p*. With *n* and *p* as centres and the same radius describe arcs, apparently coinciding with the ellipse for a short distance. Also with *c* and *d* as centres, and a radius equal to *ek* describe arcs, intersecting *cd* produced in *l* and *m*. With *l* and *m* as centres and the same radius describe arcs, apparently coinciding with the ellipse for a short distance. Having determined how far the arc will coincide with the curve on one side of the axis it should be drawn an equal length on the other side. The four arcs may be connected by the aid of the curved ruler, and the student should make the joints carefully, in order that the curve may appear continuous.

The above construction gives the radius of curvature at the vertices of the major and minor axes. In order that a larger part of the curve may be represented by arcs, make the radius a little *longer* than *na* in the one case, and a little *shorter* than *ld* in the other.

CHAPTER III.

PRACTICAL PROBLEMS.

Hitherto attention has been confined to geometrical problems. The object has been, by means of simple figures, to illustrate some of the principles which are involved in making practical working drawings. A few constructions of this kind will now be given.

PLATE VII.

Problem.—*To draw the plan and the elevation of a Pillow Block.*—Draw aa' parallel and bb' perpendicular to the ground line. Each of these lines should be drawn indefinitely. From o, the point of intersection, lay off oc and oc', each equal to one inch and a half. Through c and c' respectively, draw dd' and ee' parallel to the ground line. From c lay off cd and cd', each equal to six inches; cf and cf', each equal to five-eighths of an inch; ch and ch', each equal to one inch and a half; cj and cj', each equal to two inches and one-eighth. From the points f, f', h, h', j, j', d, and d' respectively draw $fg, f'g', hi, h'i'$, jk, jk', de and $d'e'$ perpendicular to the ground line. With each of the points l and l' as a centre, and a radius equal to one inch, describe the semi-circles mnp and $m'n'p'$. From o lay off oq and oq', each equal to four inches and three-quarters; also or and or', each equal to four inches and one-eighth. With each of the points q, q', r and r' as a centre, and a radius equal to seven-sixteenths of an inch, describe a semicircle, limited in each case by a diameter drawn perpendicular to the ground line. Connect the extremities of the diameters drawn through q and r by lines parallel to the ground line; also connect the extremities of the diameters drawn through the points q' and r' by lines parallel to the ground line. $dd'e'e$ is the plan of the block. The elevation

will be found as follows: from e, e', n, n', k, k', i, and i' respectively draw $ee_{,,} e'e_{,,} nn_{,,} n'n_{,,} kk_{,,} k'k_{,,} ii_{,}$ and $i'i_{,,}$ perpendicular to the ground line. Lay off from the ground line to $e_{,}$ a distance equal to one inch; to $c_{,,}$ a distance equal to one inch and a half; to $c_{,}$ a distance equal to four inches. From $e_{,}$ draw $e_{,}e_{,,}$; through $c_{,,}$ draw $g_{,}g_{,,}$; and through $c_{,}$ draw $n_{,}n_{,,}$ parallel to the ground line. From g and g' respectively draw $gg_{,}$ and $g'g_{,,}$ perpendicular to the ground line, intersecting $g_{,}g_{,,}$ in the points $g_{,}$ and $g_{,,}$. Lay off $i_{,}s$ and $i_{,,}s_{,}$, each equal to one inch and five-eighths. Join $sg_{,}$ and $s_{,}g_{,,}$. $e_{,}n_{,}n_{,,}e_{,,}$ is the elevation of the block. In inking the plan the following lines should be omitted, viz: the part of the line jk between the points m and p. Also the part of the line $j'k'$ between the points m' and p'. The centre lines aa' and bb' should be drawn in fine red ink; also a short red ink line perpendicular to the ground line should be drawn through each of the points q, r, l, l', r' and q', in order to locate the centres of the semicircles. In inking the elevation the line $i_{,}i_{,,}$ should be omitted. The perpendiculars to the ground line from the points $n_{,,}k_{,,}k_{,,}$ and $n_{,,}$ should be limited by the line $e_{,}e_{,,}$; and the lines $i_{,}s$ and $i_{,,}s_{,}$ should be limited respectively at the points s and $s_{,}$. The part of the line $e_{,}e_{,,}$, between the perpendiculars from $k_{,}$ and k'', should be omitted. The outlines of the holes rq and $r'q'$, being invisible, in the elevation, should be represented by broken lines. In order to complete the drawing all the dimensions should be added, a few of which are indicated in the plate. The student should be careful to write the figures neatly, a short distance from the line the length of which he wishes to indicate. As an illustration: in order to show the length of the line dd', draw a fine red ink line parallel to it, limited by the lines $ee_{,}$ and $e'e_{,,}$, and write 12″ as illustrated. Similarly the distance between the perpendiculars $i_{,}s$ and $i_{,,}s_{,}$, viz., 3″ is shown. To complete the drawing, the student should add all the dimensions which have been referred to, omitting the letters.

EXERCISE III.

To draw the plan of a square prism standing upon a square pedestal, the base of the pedestal forming with the horizontal

plane an angle of 30°. *Length of side of base of pedestal*=3″; *thickness of pedestal*=½″. *Length of side of base of prism*= 1½″; *altitude of prism*=4½″. Assume a point *a* on the ground line. From *a* draw *ab* equal to three inches, the length of the side of the base of the pedestal, forming with the ground line an angle of 30°. From *a* and *b* respectively draw *ac* and *bd* perpendicular to *ab*, that is, forming angles of 60° with the ground line, each equal to one-half inch, the thickness of the pedestal. Draw *cd* parallel to *ab*. Lay off *ce* equal to three-quarters of an inch, and *cf* equal to two inches and a quarter. From *e* and *f* respectively draw *eg* and *fh* perpendicular to *cd*, that is, forming angles of 60° with the ground line. Make *eg* equal to four inches and a half, and draw *gh* parallel to *ab*. *abdc* is the elevation of the pedestal, and *ghfe* is the elevation of the prism. Each of the lines *gh*, *ef*, *cd* and *ab* represents two lines, parallel to the vertical plane; the plans of these lines will therefore be parallel to the ground line. Each of the points *g*, *h*, *f* and *e* is the elevation of a line one inch and a half long, perpendicular to the vertical plane; the plan of each of these lines will therefore be perpendicular to the ground line, one inch and a half long. And each of the points *a*, *b*, *d* and *c*, is the elevation of a line three inches long, perpendicular to the vertical plane; the plan of each of these lines will therefore be perpendicular to the ground line, three inches long. It should be understood that the centre of the lower base of the prism coincides with the centre of the upper base of the pedestal. With these data the student is required to construct the plan.

PLATE VIII.

Problems. *To draw the elevation and the plan of a pedestal in two positions.* Draw ak' (No. 1) perpendicular to the ground line. Draw $b_{,}b_{,,}$, $c_{,}c_{,,}$, $d_{,}d_{,,}$ and $e_{,}e_{,,}$ parallel to the ground line, at distances from it respectively equal to five inches and a quarter, four inches and fifteen sixteenths, four inches and five eighths, and five eighths of an inch. Lay off $ab_{,}$ and $ab_{,,}$ each equal to two inches and five-sixteenths, and from $b_{,}$ and $b_{,,}$ respectively, draw $b_{,}f_{,}$ and $b_{,,}f_{,,}$ perpendicular to the ground line, intersecting $d_{,}d_{,,}$ in $d_{,}$ and $d_{,,}$; $e_{,}e_{,,}$ in $e_{,}$ and $e_{,,}$; and the ground line in $f_{,}$ and $f_{,,}$. From o lay off $oc_{,}$ and $oc_{,,}$ each equal to one inch and one-eighth, and draw $c_{,}g_{,}$ and $c_{,,}g_{,,}$ perpendicular to the ground line. With o as a centre, and radii respectively equal to thirteen-sixteenths of an inch, and one inch and seven-sixteenths, describe the semicircles $h_{,}h_{,,}$ and $j_{,}kj_{,,}$. Draw $l_{,}m_{,}$ and $l_{,,}m_{,,}$ each at a distance of one-quarter of an inch from the centre line ak, perpendicular to the ground line. Connect the semicircle $j_{,}kj_{,,}$ with the straight lines $d_{,}d_{,,}$, $l_{,}m_{,}$ and $l_{,,}m_{,,}$ by tangent arcs, described with a radius equal to three-eighths of an inch. Also connect the perpendiculars $l_{,}m_{,}$ and $l_{,,}m_{,,}$ with the horizontal line $e_{,}e_{,,}$ by tangent arcs, described with the same radius. In order to find the centres of these arcs, with the centre o and a radius equal to one inch and thirteen-sixteenths. (that is, three-eighths of an inch longer than the radius with which we describe the semicircle $j_{,}kj_{,,}$), describe an arc $o_{,}o_{,,}$. Draw a line parallel to $d_{,}d_{,,}$ at a distance of three-eighths of an inch from it, intersecting the arc $o_{,}o_{,,}$ in a point $o_{,}$; and a line parallel to $l_{,,}m_{,,}$ at a distance of three-eighths of an inch from it, intersecting the arc $o_{,}o_{,,}$ in a point $o_{,,}$. Also draw a line parallel to $e_{,}e_{,,}$ at a distance of three-eighths of an inch

from it, intersecting $o_{,,}o_{,,,}$ in the point $o_{,,,}$. With each of the points $o_{,,}o_{,,}$ and $o_{,,,}$ as a centre, and a radius equal to three-eighths of an inch, describe an arc, which will evidently connect the lines as required. Similar constructions should be made on the other side of the figure. From the points $d_{,}$ and $d_{,,}$ respectively, lay off $d_{,}n_{,}$ and $d_{,,}n_{,,}$, each equal to five-eighths of an inch. Connect $n_{,,}$ and $e_{,,}$ by the arc of a circle, described with a radius equal to four inches. The centre of this arc will be found as follows: with $n_{,,}$ as a centre and a radius equal to four inches, describe an arc; also with $e_{,,}$ as a centre, and the same radius, describe another arc, intersecting the former in a point o'. With this point as a centre, and a radius equal to four inches, describe the arc $n_{,,}e_{,,}$. A similar construction should be made in order to connect the points $n_{,}$ and $e_{,}$. $b_{,}b_{,,}f_{,,}f_{,}$ is the elevation of the pedestal.

The plan will be drawn as follows: draw $b_{,}b$ and $b_{,,}b'$ perpendicular to the ground line. Lay off pq and pb, respectively equal to five-eighths of an inch, and three inches. Draw pp',qq' and bb' parallel to the ground line. Prolong the lines $l_{,}m_{,}$ and $l_{,,}m_{,,}$ respectively, to l and l', and connect these lines with qq' by tangent arcs, described with a radius equal to three-eighths of an inch. Find the centres of these arcs by the method described for the point $o_{,,,}$ in the elevation. From the points $g_{,}h_{,}h_{,,}$ and $g_{,,}$ respectively, draw $g_{,}g$,$h_{,}h$,$h_{,,}h'$ and $g_{,,}g'$ perpendicular to the ground line. $pp'b'b$ is the plan of the pedestal.

To draw the plan and elevation in another position: assume the point b (No. II.) and from it draw bb' equal to bb' (No. I.) forming an angle of 30° with the ground line. From b and b' respectively, draw bp and $b'p'$ perpendicular to bb', equal to the corresponding lines in No. I, and connect p with p'. Make pq, No. II., equal to pq, No. I., and draw qq' parallel to pp'. Also lay off bg,bh,bl, etc., respectively equal to bg,bh,bl, etc., No. I., and through the points g,h,l, etc., draw parallels to bp. The plan No. II. is a copy of No. I., the plane, pp', forming with the vertical plane an angle of 30°.

The elevation, No. II, will be found as follows: prolong the lines $b_{,}b_{,,}$, $c_{,}c_{,,}$, $d_{,}d_{,,}$ and $e_{,}e_{,,}$ (No. I.) indefinitely. Erect perpendiculars from each of the points p, b, g, l, l', g', g'' and b', No. II, limiting them as indicated in the plate, that is, the perpendiculars from p, b and b', will be limited between the ground line and $e_{,}e_{,,}$, and between $d_{,}d_{,,}$ and $b_{,}b_{,,}$; the perpendiculars from l and l' will be limited between the line $e_{,}e_{,,}$ and the semi-ellipse, which is the projection of the semicircle $j_{,}kj_{,,}$; the perpendiculars from g, g' and g'' will be limited between the lines $b_{,}b_{,,}$ and $c_{,}c_{,,}$. The curve $n_{,}e_{,,}$ No. 1, represents two curves, of which rnp and $r'n'q$ are the plans. Assume any point $r_{,}$ on the curve $n_{,}e_{,}$. From $n_{,}$ and $r_{,}$ draw perpendiculars to the ground line, intersecting pp' and qq' in r, r', n and n'. Lay off pn and pr, No. II, respectively equal to pn and pr, No. I. Draw nn' and rr' parallel to pq, No. II. From n and n', No. II, respectively, draw $nn_{,,,}$ and $n'n_{,}$ perpendicular to the ground line, intersecting $d_{,,}d_{,}$ in $n_{,,,}$ and $n_{,}$. From $r_{,}$, No. I, draw a parallel to the ground line, and from r and r', No. II; erect perpendiculars intersecting this line in $r_{,,}$ and $r_{,}$. From p and q, No. II, erect perpendiculars intersecting $e_{,}e_{,,}$ in e and e'. Through the points $n_{,,,}$, $r_{,,}$ and e pass a curve. Also through the points $n_{,}$, $r_{,}$ and e' pass a curve. These curves should be drawn with the curved ruler, which should be fitted to the points, any number of which may be found in a manner similar to that described for $r_{,,}$ and $r_{,}$. The ellipses $h_{,}ih_{,,}$ and $j_{,}kj_{,,}$ will be found as follows: from k', No. II, erect a perpendicular, intersecting $j_{,}j_{,,}$ in o, the centre of the curves. From h and h' erect perpendiculars, intersecting $j_{,}j_{,,}$ in $h_{,}$ and $h_{,,}$. From the point i, No. I, draw a parallel to the ground line, intersecting $k'o$, No. II, in i. oi is the semi-major axis, and $h_{,}h_{,,}$ is the minor axis of the smaller ellipse. On these axes the semi-ellipse $h_{,}ih_{,,}$ may be constructed, as in Prob. II, Plate V. From $j_{,}$ and $j_{,,}$, No. I, draw perpendiculars intersecting bb' in j and j'. Lay off $k'j$ and $k'j'$, No. II, each equal to $k'j$ or $k'j'$, No. I. From j and j', No. II, erect perpendiculars, intersecting $j_{,}j_{,,}$ in $j_{,}$ and $j_{,,}$, the vertices of the minor axis of the larger

ellipse. The vertex of the major axis k will be found by drawing from k, No. I, a line parallel to the ground line, intersecting $k'o$, No. II, in k. It will be observed that the horizontal lines $d_{,}d_{,,}$ and the vertical lines $l_{,}m_{,}$ and $l_{,,}m_{,,}$ are connected with the larger ellipse by short curves. Also that the horizontal line $e_{,}e_{,,}$ is connected with the vertical lines $l_{,}m_{,}$ and $l_{,,}m_{,,}$ by short curves. The construction of these curves, as well as the visible portion of the curve st, is left as an exercise for the student. $eb_{,}b_{,,}f_{,,}$ is the elevation required.

PLATE IX.

Problems. *To draw the plan and the elevation of a crank in two positions.* Draw aa', No. I, perpendicular to the ground line. Lay off from the ground line to o a distance equal to two inches and eleven-sixteenths. With o as a centre, and radii respectively equal to two inches and eleven-sixteenths, two inches and a half, and one inch and nine-sixteenths, describe the circles $b_{,}b_{,,}$, $c_{,}c_{,,}$ and $d_{,}d_{,,}$. Lay off from the centre of the circles along aa' a distance equal to one inch and seven-eighths, and draw $e_{,}e_{,,}$ parallel to the ground line. Lay off from aa' to $e_{,}$ and $e_{,,}$ distances each equal to seven-sixteenths of an inch, that is, make $e_{,}e_{,,}$ equal to seven-eighths of an inch. From $e_{,}$ and $e_{,,}$ draw perpendiculars to the ground line, and produce them until they intersect the circle $d_{,}d_{,,}$ in the points $f_{,}$ and $f_{,,}$. Lay off from o to $o_{,}$ a distance equal to ten inches. With $o_{,}$ as a centre, and radii respectively equal to one inch and five-eighths, one inch and a half, fifteen-sixteenths of an inch and eleven-sixteenths of an inch, describe the circles $g_{,}g_{,,}$, $h_{,}h_{,,}$, $i_{,}i_{,,}$ and $j_{,}j_{,,}$. Draw $k_{,}k_{,,}$ parallel to the ground line and tangent to the circle $g_{,}g_{,,}$, and lay off from aa' to $k_{,}$ and $k_{,,}$ distances each equal to one inch and a quarter. Also draw $l_{,}l_{,,}$ parallel to the ground line and tangent to the circle $b_{,}b_{,,}$, and lay off from aa' to $l_{,}$ and $l_{,,}$ distances each equal to one inch and three-quarters. Draw the straight lines $k_{,}l_{,}$ and $k_{,,}l_{,,}$, and connect each of them with the circle $g_{,}g_{,,}$ by a tangent arc, described with a radius equal

to one inch. Also connect each of the lines $k_{,}l_{,}$ and $k_{,,}l_{,,}$ with the circle $b_{,}b_{,,}$ by a tangent arc, described with a radius equal to two inches and a half. The constructions for finding the centres of these tangent arcs are illustrated, and are similar to those employed in No. I., Plate VIII., to which the student's attention is directed. $g_{,}g_{,,}b_{,,}b_{,}$ is the elevation of the crank.

The plan will be found as follows: Draw cc' parallel to the ground line, and on aa' mark points at the following distances from cc', viz., $\frac{3}{16}''$, $\frac{1}{2}''$, $\frac{5}{8}''$, $1\frac{3}{8}''$, $2\frac{3}{8}''$, $2\frac{7}{8}''$, $4\frac{5}{8}''$ and $5''$. Through these points, respectively, draw mm', hh', nn', uu', bb', pp', qq' and ii' parallel to the ground line. The length of these lines will be determined as follows: tangent to each circle in the elevation, draw two lines perpendicular to the ground line, intersecting the plan in its extremities. A careful examination of the Plate will enable the student to see clearly the connection between the plan and elevation of each circle. The line j is connected with the points p and q by quadrants, described with a radius equal to one-quarter of an inch, and the centres of these quadrants are on the line pq. Similarly the line $j_{,}$ is connected with the points p' and q'. The points c and m are connected by a quadrant, described with a radius equal to three-sixteenths of an inch, and the centre of this quadrant is at the intersection of the lines bm and $c'c$. The quadrant $c'm'$ is constructed in a similar manner. The points h and n are connected by a quadrant, described with a radius equal to one-eighth of an inch, and the centre of this quadrant is at the intersection of the lines gn and $h'h$. The quadrant $h'n'$ is drawn similarly. With o', the point of intersection of pp' with aa', as a centre, and a radius $o'i$ describe the arc ii'. From $e_{,}$ and $e_{,,}$ draw perpendiculars intersecting cc' in t and t', and bb' in e and e'. $cc'i'i$ is the plan of the crank.

Draw the plan and elevation in another position as follows: make the plan, No. II, a copy of the plan No. I, and make those lines, which in No. I. are parallel and perpendicular to the ground line, form angles of 45° with the ground line in No. II.

The elevation will be found as follows: from $o_{\prime}$, No. I, draw $o_{\prime}o_{\prime}$ parallel to the ground line. This line will contain the centres of the ellipses which are the projections of the circles ii', qq', pp', ss', gg' and nn'. Also from o, draw oo parallel to the ground line. This line will contain the centres of the ellipses which are the projections of the circles bb', mm', dd' and rr'. The major axis of the ellipse will, in each case, be perpendicular to the ground line, and equal to the diameter of the circle of which the ellipse is the projection. It is needless to give the construction for each one, as the method of making it is illustrated in Prob. II, Plate V. The points $k_{\prime}$, $k_{\prime\prime}$, $l_{\prime}$ and $l_{\prime\prime}$, No. I, will in the plan be found in the line bb'. Transferring these points by measurement to the corresponding position in No. II, their elevations will be found as illustrated. Join $k_{\prime}\,l_{\prime}$ and $k_{\prime\prime}\,l_{\prime\prime}$. From k draw kv perpendicular to bb', intersecting uu' in v. From v draw $vv_{\prime}$ perpendicular to the ground line, intersecting $k_{\prime}k_{\prime\prime}$ in $v_{\prime}$. Through $v_{\prime}$ draw $v_{\prime}w_{\prime}$ parallel to $k_{\prime}l_{\prime}$. From $e_{\prime}$ and $f_{\prime}$, No. I, draw $e_{\prime}e_{\prime\prime}$ and $f_{\prime}f_{\prime\prime}$ parallel to the ground line. From the points e and e' in the plan No. II, draw perpendiculars to the ground line, intersecting $e_{\prime}e_{\prime\prime}$ in $e_{\prime}$ and $e_{\prime\prime}$, and $f_{\prime}f_{\prime\prime}$ in $f_{\prime}$ and $f_{\prime\prime}$. $e_{\prime}\,e_{\prime\prime}\,f_{\prime\prime}\,f_{\prime}$ is the visible part of the key-way. The student should construct the tangent curves connecting the straight lines $k_{\prime}l_{\prime}$ and $k_{\prime\prime}l_{\prime\prime}$ with the ellipses.

Ex. IV.—*To draw the plan of an hexagonal prism standing upon an octagonal prism; the planes of the bases forming with the horizontal plane angles of* 30°. Let the dimensions and position of the solid, relative to the vertical plane, be the same as in Prob. IV, Plate I. That is, assume the elevation $eiq''v''$ a copy of $eiq''v''$, Plate I. In Plate I. the planes of the bases are horizontal; hence, their elevations are parallel to the ground line. In the exercise they will form with the ground line angles of 30°. Also in Plate I. the elevations of the edges, which are perpendicular to the bases, are perpendicular to the ground line. In the exercise they will form with the ground line angles of 60°. If it be remembered that in the exercise the position of the figure

relative to the vertical plane is the same as that in Plate I., it will be apparent the plans of the edges, which are perpendicular to the bases, will be parallel to the ground line, and their distances from it will be found in the plan, Plate I. With these data the student is required to make the construction.

PLATE X.

Problems. *To draw the elevation and the plan of a crank shaft in two positions.* First draw the elevation No. I. when the axes of the shaft and crank-pin are parallel to the vertical plane, the crank being in a vertical position. Let the diameter of the shaft be two inches, the diameter of the pin one inch and three-quarters, and the distance between the axes of the shaft and the pin five inches. Draw $aa_{,}$ the elevation of the axis parallel to the ground line, at a distance from it equal to one inch, that is, one-half the diameter of the shaft. Draw $bb_{,}$ the elevation of the axis of the pin parallel to the ground line, at a distance from $aa_{,}$ equal to five inches. Draw $cc_{,}$ perpendicular to the ground line. Draw dg parallel to the ground line, at a distance of one inch from $bb_{,}$. From c, the point in which dg intersects $cc_{,}$, lay off cd equal to one inch and three-quarters, cf equal to two inches and a half, and cg equal to four inches and a quarter. From d and g draw perpendiculars to the ground line, limited by the line $aa_{,}$ respectively in the points $d_{,}$ and $g_{,}$. Also from f draw $ff_{,}$ perpendicular to the ground line.

Draw hj parallel to the ground line, at a distance from it equal to two inches, the diameter of the shaft. On hj from $dd_{,}$ lay off to h a distance equal to three inches, and draw $hh_{,}$ perpendicular to the ground line, also on hj from $gg_{,}$ lay off to j a distance equal to three inches, and draw $jj_{,}$ perpendicular to the ground line. From o, the point in which $bb_{,}$ intersects $cc_{,}$ lay off oe and $oe_{,}$ each equal to seven-eighths of an inch, that is, one half the diameter of the pin, and draw ei and $e_{,}i_{,}$ parallel to the ground line. $h_{,}dgj_{,}$ is the required elevation,

To construct the plan, draw $a'a''$ parallel to the ground line. Draw kl and $k'l'$, parallel to $a'a''$, each at a distance from it equal to one inch, that is, one half the diameter of the shaft. Produce the perpendiculars $hh_{,}, dd_{,}, cc_{,}, ff_{,} qq_{,}$ and $jj_{,}$ intersecting the lines kl and $k_{,}l_{,}$ in the points $k, k', m, m', n, n', p, p', q, q', l,$ and l'. Draw rs and $r's'$ parallel to $a'a''$, each at a distance from it equal to seven-eighths of an inch, that is, one-half the diameter of the crank pin. $kll'k'$ is the required plan.

Now draw the plan and elevation (No. II.) when the axes of the shaft and pin are parallel to the horizontal plane, and form with the vertical plane angles of 30°, the crank being in a vertical position. Draw the plan, making it a copy of the plan No. I, drawing all those lines which in No. I are parallel to the ground line, forming angles of 30° with the ground line; also all those lines which in No. I are perpendicular to the ground line, forming angles of 60° with the ground line, that is, make one set of lines perpendicular to the other set.

In drawing the elevation, we suppose that the shaft is resting upon the horizontal plane as in No. I. The axes of the shaft and pin will therefore be at the same distance from the horizontal plane as in No. I, that is, their elevations will be in the prolongations of these lines, as illustrated in the plate. These broken lines will contain all the centres of the ellipses which are the projections of the circles. The major axes of the ellipses will be perpendicular to the ground line, drawn from the points in the plan in which the plan of the axes intersects the lines which are perpendicular to it, and in each case the major axis will be equal to the diameter of the circle of which the ellipse is the projection. With these suggestions, and with the aid of the illustration, the student will have little difficulty in constructing the elevation.

CHAPTER IV.

ISOMETRIC DRAWING.

PLATE XI:

A series of exercises on Isometric Drawing, sometimes known as *Isometric Perspective*, will now be given. By this kind of drawing a view of the object to be represented is produced, which gives to the observer a better idea of its form than that which is suggested by the plan and elevation.

The simplest way to approach the subject will be by making a projection of a cube as follows: assume a point *o*, Fig. I, and draw *oa* of any convenient length, perpendicular to the ground line *GL*. With *o* as a centre, and the radius *oa*, describe the circle *a b c d e f*, and within this circle inscribe the regular hexagon *a b c d e f*. Join *oc* and *oe*. The lines *ed*, *oc* and *ab* are parallel and equal to each other; the lines *cd*, *oe* and *af* are parallel and equal to each other; and the lines *cb*, *oa* and *ef* are parallel and equal to each other. From *b*, *d* and *f* draw broken lines respectively parallel to *af*, *ef* and *ab*, intersecting at the point *o*. Erase the circle, and we shall have the projection of a cube when a great diagonal is projected in the point *o*, that is, when this diagonal is perpendicular to the vertical plane, all the edges being projected in lines of equal length. Such a representation of a cube is called its *isometric projection*. Since the great diagonal *o* is perpendicular to the vertical plane, the adjacent edges *oa*, *oc* and *oe* form with the vertical plane equal angles, and are projected in lines of equal length, as we have illustrated. Also, since the length of the edges *oa*, *oc* and *oe* are equal, the points *a*, *c* and *e* are equidistant from the vertical plane, and the lines joining these points will be parallel to the vertical plane, and will be projected upon it in their true length. Join *ac*, *ce* and *ea*. Now, since the line *ec* is projected in its true length, it is the length of the diagonal of the

square which is projected in the parallelogram *ocde*. In order to find the true length of the side of this square, from the points *c* and *e* draw straight lines, forming with *ec* angles of 45°, intersecting in the point *d'*. Similar lines constructed on the other side of the diagonal would complete the square, but for the purpose the lines which are drawn are all that are necessary.

To construct the projections of the circle inscribed in the visible faces of the cube, the major axis of the ellipse in each case will be the projection of that diameter of the circle which is parallel to the vertical plane, and will be contained in one or other of the lines connecting the points *a*, *c* and *e*. Now, since the diameter of the inscribed circle will be equal to the length of the side of the square, the axes will be found as follows: bisect *ac* in *h* and lay off *hi* and *hk*, each equal to one-half the length of the line *cd'* or *ed'*; *ik* will be the major axis of the ellipse. In order to construct the minor axis, conceive that a square is inscribed within the circle, and that *i* and *k* are the projections of two of the angular points of this square. Draw *il* and *km* parallel to *oc*, and draw *im* and *kl* parallel to *oa*. *il* intersects in *kl* in *l*, and *im* intersects *km* in *m*. Join *lm*, which will be perpendicular to *ik*. *lm* will be the minor axis of the ellipse. On *ik* and *lm*, as the axes, construct the ellipse *i m k l*, which will be tangent to the lines *oc*, *cb*, *ba* and *ao* at their middle points. Similarly construct the ellipses, which are the projections of the circles inscribed in the faces *o c d e* and *o e f a*. It will be observed that these ellipses are equal to each other in every respect, that the major axes produced form the equilateral triangle *e c a*, that the line *ec* is parallel to the ground line, and that the lines *ea* and *ca* form with the ground line angles of 60°. These three ellipses are the projections of circles in the different positions which they *generally* occupy in practical problems. The illustrations have therefore been given complete for future reference.

In the drawing there are three sets of parallel lines, one set being perpendicular to the ground line, the remaining lines forming with the ground line angles of 30°. A drawing of this kind may be made very conveniently with the

aid of the 30° triangle applied to the T square. Hereafter it will be unnecessary to draw the ground line which has been introduced for reference. It will be understood that perpendicular lines are drawn perpendicular to the edge of the blade of the T square, and that 30° lines form with this edge angles of 30°; that the major axis of an ellipse will be ruled along this edge, as in the ellipse inscribed in the face of the cube, *ocde*, or that the major axis will form with this edge an angle of 60°, as in either of the other cases.

In the illustration that has been given we have not been confined to any particular dimensions. But if it be required to draw the isometric projection of a cube of any given dimensions, it will be necessary to construct a scale from which to take measurements. It is evident in Fig. I that each edge of the cube is projected in a line which is shorter than its true length. Now, if we can ascertain the true length of the edge which is projected in the line *cd*, we may then construct a scale whose length shall bear to the length of the ordinary scale the same proportion which the line *cd* bears to the line of which it is the projection. The length of this line will be found as follows: We have seen that the true length of the diagonal of the face is projected in the line *ec*, and that the lines *cd'* and *ed'*, forming angles of 45° with the diagonal, determine two sides of the square. Conceive this square to be revolved about its diagonal *ec* into a position when the point *d'* is projected in *d*. In that position the line *cd'* is projected in *cd*, and the line *ed'* in *ed*. Hence, *cd'* is the true length of the line of which *cd* is the projection, and since the line joining *d'* with *d* is perpendicular to *ec*, it is evident that if we lay off from *c* to *s'* a distance equal to two inches, and from *s'* draw *s's* perpendicular to *ec*, intersecting *cd* in *s*, and *cs*, will be the projection of a line two inches long; that is, when the square is revolved into the position *ocde*, the point *s'* will be projected in *s*. From this it will be plain that the following will be a correct construction of the required scale: at any point *a*, Fig. II, draw *ab* and *ab'* forming with *ac* respectively, angles of 45° and 30°. Lay off *ab* equal to the length of the

required scale, we will say twelve inches. From b draw be perpendicular to ac, intersecting ab' in the point b'. ab' will be the length of the required scale. In order to compare this construction with that of Fig. I, complete the square $abcd$, the length of whose side is twelve inches. If we conceive this square to be revolved about its diagonal ac until the sides are projected in the lines $ab', b'c, cd'$ and $d'a$, which form with ac angles of 30°, we shall have the projections of the sides of a square twelve inches long, and the length of either of these lines will be the length of the required scale. A scale which bears to the ordinary scale the same proportion which ab' bears to ab is called an *isometric scale.* We have divided the ordinary scale cb into twelve equal parts, that is, into twelve inches, and the corresponding points of division on the isometric scale cb' are determined by drawing perpendiculars to the line ac, as illustrated.

PLATE XII.

It has been shown (Fig. II., Plate XI.) how to find the length of an isometric scale.

Before drawing the problems of this plate, the student should construct an ordinary scale, No. I, twelve inches long, and also an isometric scale, No. 2, of the same length, that is, the length of No 1 should be made equal to that of bc—Fig. II., Plate XI., and the length of No. II., should be made equal to that of $b'c$ in the same figure. It is understood that these constructions are to be drawn full size. Having made these scales, we shall be ready to construct the following:

Prob. I.—*To draw the isometric projection of a rectangular block, six inches long, three inches wide, and one inch thick.*

The elevation of a block of these dimensions is a rectangle $abcd$, whose length ab is equal to six inches, the given length, and ad equal to one inch, the thickness of the block. The plan is a rectangle $a'b'e'f'$, whose width $a'f'$ is equal to three inches, the given width.

To draw the isometric projection, assume a point $a_{\prime}$, and draw a perpendicular $a_{\prime}a_{\prime}d_{\prime}$. Also draw 30° lines $a_{\prime}b_{\prime}$ and $a_{\prime}f_{\prime}$;

$a_{,}d_{,}$ will be perpendicular to the edge of the T square, and $a_{,}b_{,}$ and $a_{,}f_{,}$ will form angles of 30° with this edge, that is, these lines will form with each other angles of 120°. Now it is evident that if we make $a_{,}d_{,}$ one inch long by the shorter or isometric scale, and lay off $a_{,}g_{,}$ and $a_{,}h_{,}$ each equal to $a_{,}d_{,}$ and complete the regular hexagon $d_{,}h_{,}g_{,}$, as in Fig. I., Plate XI., we shall have the isometric projection of a cube the length of whose edge is one inch; and it is also evident that if we prolong the line $a_{,}g_{,}$ to $b_{,}$ making the length of $a_{,}b_{,}$ equal to six times the length of $a_{,}g_{,}$, that is, equal to six inches by the isometric scale, we shall have the isometric projection of a line six inches long; and also, if we prolong $a_{,}h_{,}$ to $f_{,}$, making the length of $a_{,}f_{,}$ equal to three times the length of $a_{,}h_{,}$, that is, equal to three inches by the isometric scale, we shall have the isometric projection of a line three inches long; and since $a_{,}d_{,}$ is the isometric projection of a line one inch in length, and the remaining edges of the block are parallel to one or other of the lines $a_{,}d_{,}$, $a_{,}b_{,}$ or $a_{,}f_{,}$, the following construction will complete the drawing: from $b_{,}$ and $f_{,}$, respectively, draw perpendiculars $b_{,}c_{,}$ and $f_{,}l_{,}$, that is, parallel to $a_{,}d_{,}$; from $f_{,}$ and $d_{,}$ respectively, draw $f_{,}e_{,}$ and $d_{,}c_{,}$ parallel to $a_{,}b_{,}$; and from $b_{,}$ and $d_{,}$, respectively, draw $b_{,}e_{,}$ and $d_{,}l_{,}$ parallel to $a_{,}f_{,}$. $f_{,}e_{,}b_{,}c_{,}d_{,}l_{,}$ is the required projection.

The construction may be described in a few words as follows: Assume a point $a_{,}$. From $a_{,}$ draw the perpendicular $a_{,}d_{,}$ and the 30° lines $a_{,}b_{,}$ and $a_{,}f_{,}$ indefinitely; lay off by the isometric scale $a_{,}d_{,}$ equal to one inch, $a_{,}f_{,}$ equal to three inches, and $a_{,}b_{,}$ equal to six inches. Complete the figure by drawing parallels to these lines, from the points $d_{,}$, $b_{,}$ and $f_{,}$. The broken lines represent the invisible edges.

Prob. II.—*To draw the isometric projection of a block whose plan and elevation are given.*—Assume the plan as follows: draw $a'b'$ and $a'c'$ perpendicular to each other, respectively equal to five inches and three-eighths, and to three inches. Lay off $a'd'$ equal to three inches and a half, and draw $d'e'$ perpendicular to $a'b'$, equal to two inches and five-eighths.

Join $b'e'$ by a straight line, and connect the points c' and e' by the arc of a circle described with a radius equal to four inches. $a'b'e'c'$ is the plan of the solid. The elevation is the rectangle $abfg$, the length ab being equal $a'b'$, and ag being equal to one inch and a quarter.

To draw the isometric projection, assume a point $a_{\prime}$ and draw by the isometric scale the 30° lines $a_{\prime}b_{\prime}$ and $a_{\prime}c_{\prime}$ respectively, equal to five inches and three-eighths, and three inches, and the perpendicular $a_{\prime}g_{\prime}$ equal to one inch and a quarter. Draw the 30° lines $g_{\prime}f_{\prime}$ and $g_{\prime}h_{\prime}$ and the perpendiculars $b_{\prime}f_{\prime}$ and $c_{\prime}h_{\prime}$. Lay off $a_{\prime}d_{\prime}$ equal to three inches and a half by the isometric scale, and draw the 30° line $d_{\prime}e_{\prime}$ equal to two inches and five-eighths. Join $b_{\prime}e_{\prime}$. The projection of the curve $c'e'$ will be found as follows: assume a series of points, i',k', &c., &c., upon it. From these points draw perpendiculars to $a'b'$, viz.: $i'i''$ and $k'k''$, &c., &c. Measure the distance $a'i''$ by the ordinary scale, and lay off this distance taken from the isometric scale from $a_{\prime}$ to $i_{\prime\prime}$. From $i_{\prime\prime}$ draw a 30° line. Measure the distance $i''i'$ by the ordinary scale, and lay off this distance taken from the isometric scale from $i_{\prime\prime}$ to $i_{\prime}$; $i_{\prime}$ is one point of the curve. Similarly find the point $k_{\prime}$ and having found a sufficient number of points rule a curve $c_{\prime}i_{\prime}k_{\prime}e_{\prime}$ through them. The number of points which it is necessary to construct depends entirely upon the length of the curve whose projection is required. It is not difficult to decide the number, and this must be left to the discretion of the draughtsman in any given drawing. We have not drawn the invisible lines as in the preceding problem, in order not to confuse the illustrations which have been given for constructing the points; but after drawing the upper curve the construction lines may be erased, and the lower curve and straight lines may be drawn.

All the straight lines in the cube, Fig. I, Plate XI, are called *isometric lines;* on any one of them a dimension may be laid off by the isometric scale. Also all the lines in the isometric projection, Prob. I., Plate XII, are isometric lines, every line being parallel to an edge of the cube. Now in Prob II. of this

plate, while the lines $a_{\prime}b_{\prime}, a_{\prime}c_{\prime}, a_{\prime}g_{\prime}$, and all the lines which are parallel to one or other of these, are isometric lines, those which are not parallel to one or other of these, as $b_{\prime}e_{\prime}$, are called *non-isometric lines*. It must be distinctly understood that, in constructing an isometric projection, we must first ascertain the dimensions of the object to be represented by the ordinary scale, and these measurements must be made in three directions perpendicular to each other; we then lay off these dimensions, taken from the isometric scale, on lines forming with each other angles of 120° as we have illustrated; but in order to draw the projection of a non-isometric line, as $b'e'$, one or both of its extremities must be located by isometric lines.

PLATE XIII.

Prob. I.—*To draw the isometric projection of a pyramid standing upon a pedestal.*—The dimensions may be assumed as follows:—draw the rectangle $a'b'e'd'$, the plan of the pedestal, four inches long by two inches wide, and the rectangle $abef$, the elevation of the pedestal, four inches long by one-half inch. The plan of the base of the pyramid will be the rectangle $g'h'i'k'$, three inches long by one inch and a half wide, the distance between the lines $a'd'$ and $k'g'$ being equal to one-half inch, and the distance between the lines $a'b'$ and $g'h'$ being equal to one-quarter of an inch. To assume the plan of the vertex o', lay off $g'l'$ equal to two inches, draw $l'o'$ perpendicular to $g'h'$ equal to one inch. Join $o'g', o'h', o'i'$ and $o'k'$. These lines are the plans of the edges of the pyramid. The elevation of the pyramid will be found by projecting the points g' and h' respectively in g and h, and the point o' in o, marking it at a distance of two inches and a half from ab, this being the altitude of the pyramid. To draw the isometric projection: assume a point $a_{\prime}$ and construct the projection of the pedestal as in Prob. I., Plate XII., the dimensions being four inches by two inches by one-half inch, and each of these dimensions will be laid off from the isometric scale. It must be remembered that *all dimensions on isometric lines should be*

laid off from the isometric scale. In order to find one point, $g_{,}$, of the base of the pyramid, produce $k'g'$ in the plan to m'. $d'm'$ is equal to one-half inch, and $g'm'$ is equal to one quarter of an inch. Make $a_{,}m_{,}$ equal to one-half inch, and draw the 30° line $m_{,}g_{,}$ equal to one quarter of an inch. Draw the projection of the rectangle three inches by one inch and a half, the sides of which will be 30° lines, as illustrated. To find the projection of the vertex lay off $g_{,}l_{,}$ equal to two inches, and draw the 30° line $l_{,}o_{,}$ equal to one inch. These distances correspond with those in the plan. From $o_{,}$ draw the perpendicular $o_{,}o_{,,}$ equal to two inches and a half, which corresponds with the altitude given in the elevation. Join $o_{,,}$ with each of the angular points of the base of the pyramid. $o_{,,}b_{,}f_{,}d_{,}$ is the required projection.

Prob. II.—*To draw the isometric projection of an upright crosz.* The dimensions may be assumed as follows: draw the rectangle $a'b'c'd'$, the plan of the cross, three inches and a quarter long by one-half inch. The elevation of this part will be the rectangle *abef*, three inches aud a quarter long by threcquarters of an inch. Lay off *ag* equal to one inch and a quarter. Draw *gh* perpendicular to *ab* one inch and a quarter long, and complete the rectangle *hikl*, four inches and a half by three-quarters of an inch. The lines in the plan, $m'h'$ and $n'i'$, are respectively the prolongation of the lines *hl* and *ik*. To draw the isometric prcjection: assume a point $a_{,}$, and construct the projection of the cross piece as in Prob. I., Plate XII., the dimensions being three inches and a quarter by one-half inch by three-quarters of an inch. An examination of the drawing will show on which line each dimension is laid off. Lay off $a_{,}g_{,}$ equal to one inch and a quarter and draw the perpendicular $g_{,}h_{,}$ equal to one inch and a quarter. Taking $h_{,}$ as the projection of the angular point h' of the rectangle $h'i'n'm'$ in the plan, draw the projection of this rectangle three-quarters of an inch by one-half inch, and from each of the angular points draw a perpendicular four inches and a half long, the altitude of the cross, which is shown in the elevation. Connect the extremities of these perpendicu-

lars by lines which form the projection of the lower end of the upright piece, that is, of a rectangle three-quarters of an inch by one-half inch. $n_{,}b_{,}k_{,}d_{,}$ is the required projection. The visible edges are drawn full, and the invisible edges broken. It will be observed that all the lines in this figure are isometric lines.

In Prob. I. all the lines which are the projections of the edges of the pedestal, and of the sides of the base of the pyramid are isometric lines, while those which are the projections of the edges connecting the vertex with the angular points of the base are non-isometric lines.

The attention of the student is particularly called to the construction for finding the projection of the vertex, as it is the key to the whole subject of isometric projection. $o_{,,}$ is found by means of three measurements, viz.: $g_{,}l_{,}$, $l_{,}o_{,}$ and $o_{,}o_{,,}$, the situation of this point being located by the plan o' and the elevation o, from each of which we can take two measurements.

Ex. V.—*To draw the elevation of a hollow cylinder whose axis is parallel to the horizontal plane, and which forms with the vertical plane an angle of* 45°. Let the rectangle $a'b'c'd'$, six inches long by three inches and a half, be the plan of the cylinder. The sides of this rectangle form with the ground line angles of 45°. Draw $e'f'$ parallel to $a'b'$, at a distance from it equal to one-half inch. Also draw $g'h$ parallel to $c'd'$, at a distance from it equal to one-half inch, and draw $i'k'$, the plan of the axis. The lines $a'd'$ and $b'c'$ are the plans of the bases of the cylinder, and the distance between the broken lines $e'f'$ and $g'h'$ measures the diameter of the hole, the thickness of the material being one-half inch.

PLATE XIV.

Problem.—*To draw the isometric projection of a solid bounded by plane and curved surfaces.* Let the plan and elevation of the solid be assumed as follows: draw the rectangle $a'b'c'd'$, the plan of the pedestal, four inches by two inches. Also draw the rectangle $e'f'g'h'$, the plan of the upper horizontal plane

surface, one inch by one quarter of an inch. The sides $e'f'$ and $g'h'$ should be parallel to $a'b'$ and $c'd'$, and at distances from these lines respectively of seven-eighths of an inch. Also $e'h'$ and $f'g'$ should be parallel to $a'd'$ and $b'c'$, and at distances from these lines respectively of one inch and a half. Join $e'a'.f'b',g'c'$ and $h'd'$. $e'f'g'h'$—$a'b'c'd'$ is the plan of the solid. To construct the elevation: draw the rectangle *abik* four inches by one-half inch; draw *ef* one inch long parallel to *ki*, at a distance from it equal to four inches. The extremities of the line *ef* will be found by erecting perpendiculars from e' and f', intersecting *ef* in the points *e* and *f*. Connect the points *e* and *a* by an arc of a circle, described with a radius equal to five inches. Also connect the points *f* and *b* by a similar arc. *efik* is the elevation.

To construct the isometric projection: assume a point $a_{,}$ and draw the projection of the pedestal, four inches by two inches by one-half inch, as in Prob. I., Plate XII. To find the point $e_{,}$ the projection of one point of the rectangle, the outline of the upper plane surface: produce $h'e'$ until it intersects $a'b'$ in l'. Also draw *el* perpendicular to *ab*. The distance from a' to l' is one inch and a half; lay off this distance from $a_{,}$ to $l_{,}$ and draw the 30° line $l_{,}m_{,}$. The distance from l' to e' is seven-eighths of an inch; lay off this distance from $l_{,}$ to $m_{,}$ and draw the perpendicular $m_{,}e_{,}$. The distance from *l* to *e* is three inches and a half; lay off this distance from $m_{,}$ to $e_{,}$ and construct the projection of the rectangle $e_{,}f_{,}g_{,}h_{,}$ one inch by one-quarter of an inch. The points $a_{,},b_{,},c_{,},d_{,},e_{,},f_{,},g_{,}$ and $h_{,}$ are the projections of the extremities of the curves. Intermediate points may be found as follows: suppose a plane passed through the solid, parallel to the base of the pedestal. This plane will cut from it a rectangle, the elevation of which will be a line *np* drawn parallel to *ab*, and the plan a rectangle $n'p'q'r'$, the sides $n'p'$ and $q'r'$ being parallel to $a'b'$, and the sides $n'r'$ and $p'q'$ parallel to $a'd'$. The length $n'p'$ is equal to *np*, and the width is determined by dropping perpendiculars from the points *n* and *p*, limiting them between the lines $e'a',f'b',g'c'$ and $h'd'$.

Find the projection of this rectangle $n_{,}p_{,}q_{,}r_{,}$, by a process similar to that employed for finding the projection of the upper one, the construction of each being fully illustrated. The four angular points are points in the projections of the curves, any number of which may be found by similar constructions, which should be taken near together in order that the curves may be accurately traced.

PLATE XV.

Problem. *To draw the isometric projection of a Pillow Block.*

Let the dimensions be those which are given in Plate VII. The construction will be made as follows: draw the projection of the pedestal twelve inches by three inches by one inch, as in Prob I., Plate XII. Bisect *ac* in *b*, and erect the perpendicular *bd* equal to four inches. Through *d* draw the 30° line *ef*, and lay off *de* and *df*, each equal to two inches and one-eighth. Draw the perpendiculars *eg* and *fh*. Lay off *di* and *dj*, each equal to one inch and a half, and draw the perpendiculars *ik* and *jl*, each equal to one inch and five-eighths. Lay off *bm* equal to one inch and a half, and through *m* draw the 30° line *no* one inch and a quarter long, that is, lay off *mn* and *mo*, each equal to five-eighths of an inch. Join *kn* and *lo*, and from each of the points *f,j,l,o,i,e* and *g* draw 30° lines, each three inches long. From $j_{,}$ draw the perpendicular $j_{,}l_{,}$ intersecting $ll_{,}$ in $l_{,}$; and from $l_{,}$ draw a parallel to *lo* as far as it is visible. Bisect $ff_{,}$ at the point ${}_{,}b$, and draw the 30° line ${}_{,}bq$. Make ${}_{,}bt_{,,}{}_{,}bq$ and ${}_{,}bt$ each equal to one inch by the *isometric* scale. Draw ${}_{,}bs$ one inch long by the ordinary scale, parallel to the ground line. Draw the perpendicular ${}_{,}bs_{,}$ and from *s* draw the 30° line $ss_{,}$ intersecting ${}_{,}bs_{,}$ in $s_{,}$. ${}_{,}bs$ is the semi-major axis, and ${}_{,}bs_{,}$ the semi-minor axis of an ellipse, one-half of which will be the projection of the semi-circle $m'n'p'$, and this curve may be drawn as in Fig. I., Plate XI., to which the student's attention is called. The major axis is equal to the diameter of the circle of which the ellipse is the projection, and its length in this particular case is laid off on the line

corresponding to *ec* drawn parallel to the ground line. From *s* draw a perpendiclar as far as it is visible. Similarly construct the semi-ellipse $v_{,}u_{,}uv$ and the visible part of the curve $w_{,}wx$. Draw the perpendiculars $u_{,}w_{,}$ and *vx*. By similar constructions we may find the projections of the holes *rq* and $r'q'$, as follows: draw the centre line *yz*. Lay off $yo_{,}$ and $yo_{,,}$ equal respectively to one inch and a quarter, and one inch and seven-eighths. The point $o_{,}$ is the centre of a semi-ellipse, which will be drawn in a manner similar to the construction for the curve $v_{,}u_{,}uv$; and the point $o_{,,}$ is the centre of a semi-ellipse, which will be drawn in a manner similar to the construction for the curve $t_{,}s_{,}qst$. The major axis in each case will be equal to seven-eighths of an inch by the ordinary scale; and seven-sixteenths of an inch by the isometric scale will be laid off from the centre on each of the 30° lines to determine other points of the curves. The vertex of the minor axis will be found as before illustrated. These semi-ellipses are connected by 30° lines which are tangent to the curves. Similarly the projection of the outline of the hole at the other end is found.

Ex. VI. *To draw the elevation of a solid whose plan is given.*

Let the plan be a rectangle $a'a''h''h'$, a copy of the plan, Prob. IV., Plate II., the lines $a'a''$ and $h'h''$ forming angles of 60°, and the lines $a'h'$ and $a''h''$ forming angles of 30° with the ground line. Draw all the lines, both visible and invisible, and find the elevation, it being understood that the distances from the horizontal edges to the horizontal plane are equal respectively to the distances from the points *a,c,e,f,* etc., to the ground line.

Ex. VII. Draw the isometric projection of a pedestal; dimensions given, Plate VIII., No. I.

Ex. VIII. Draw the isometric projection of a crank; dimensions given, Plate IX., No. I.

Ex. IX. Draw the isometric projection of a crank shaft; dimensions given, Plate X., No. I.

Remark. In these exercises the student will have an opportunity of drawing the isometric projection of a circle in different positions. The major axis of the ellipse will be equal to the diameter of the circle of which it is the projection, and its direction will be the same as in one or other of the three cases, viz.: *ac, ce* or *ea,* Fig. I., Plate XI., to which attention is especially directed.

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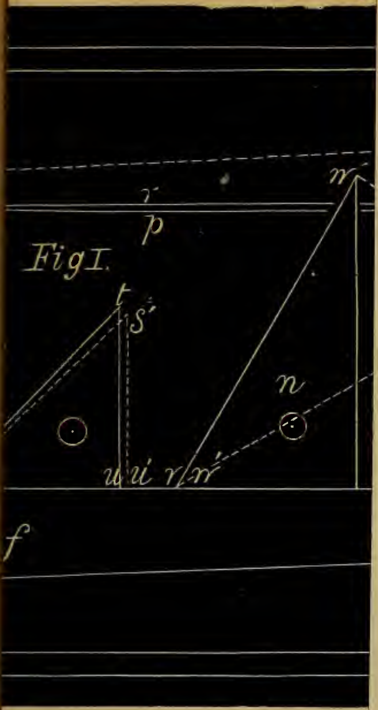

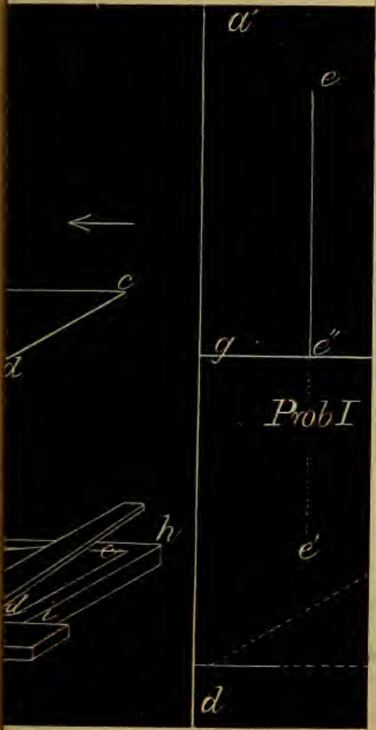

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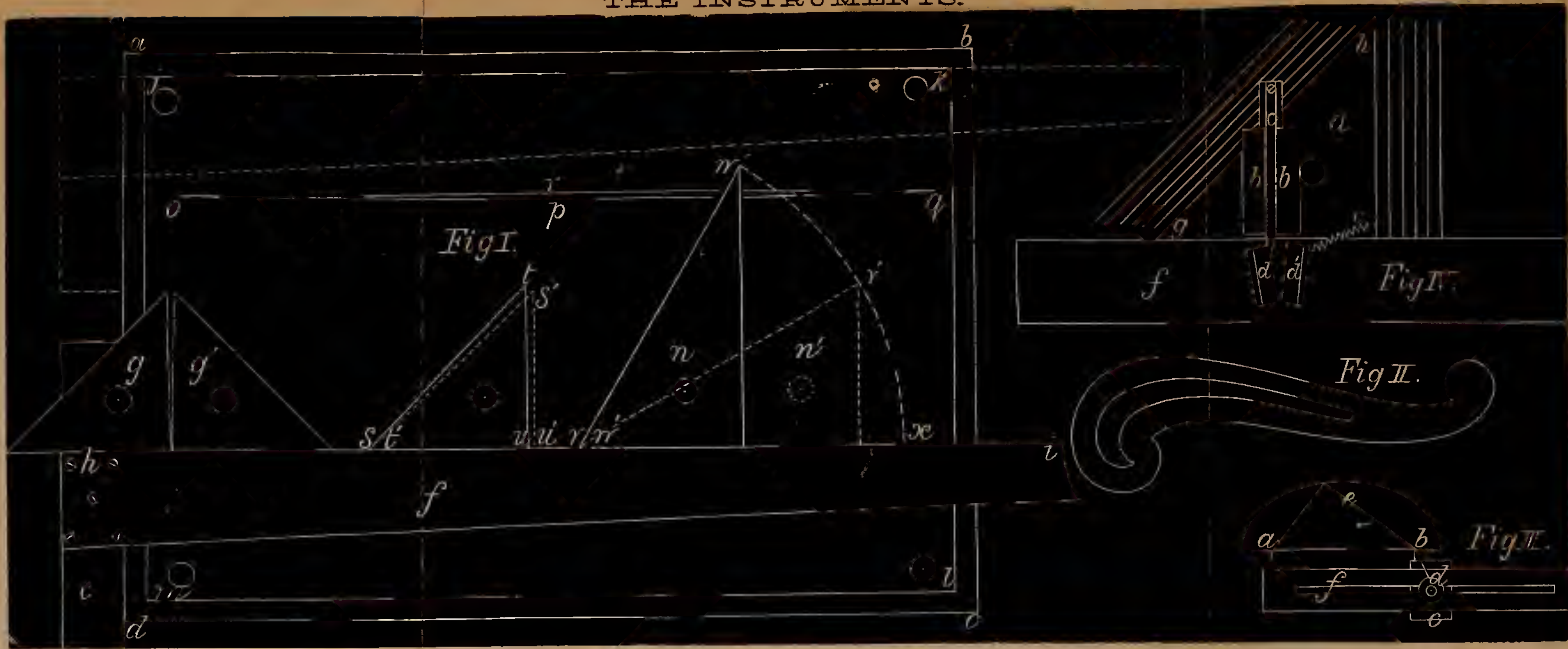
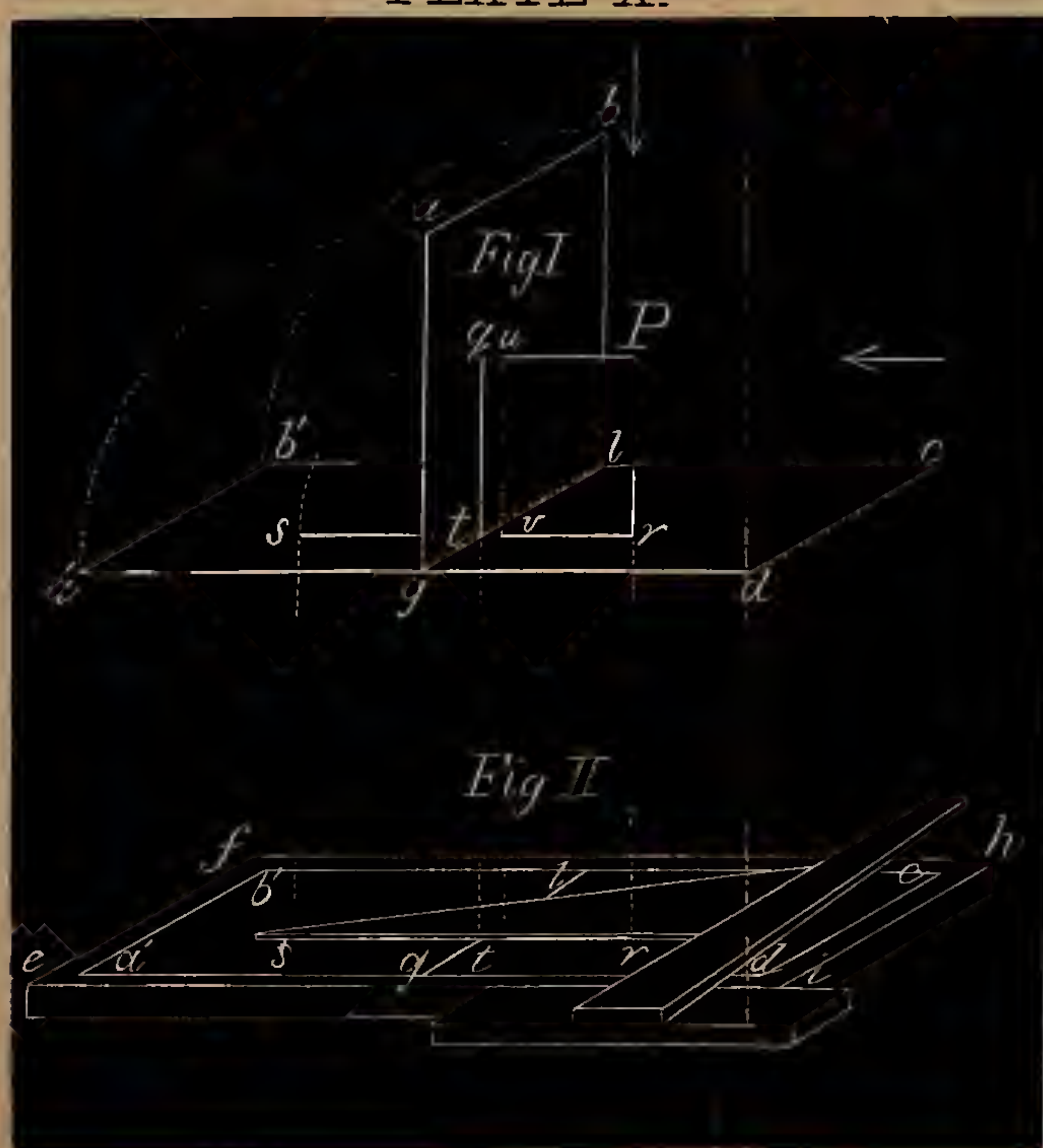
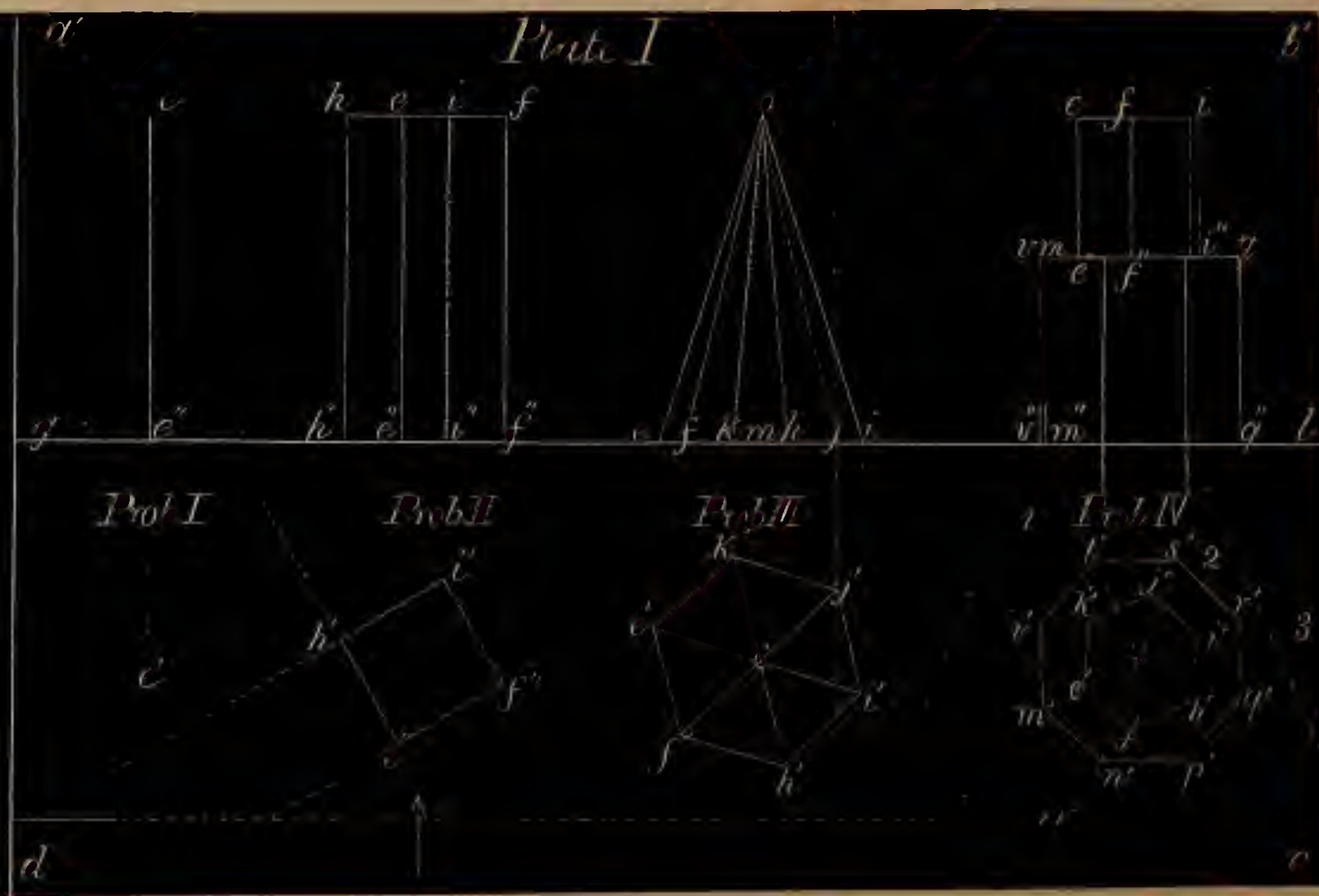
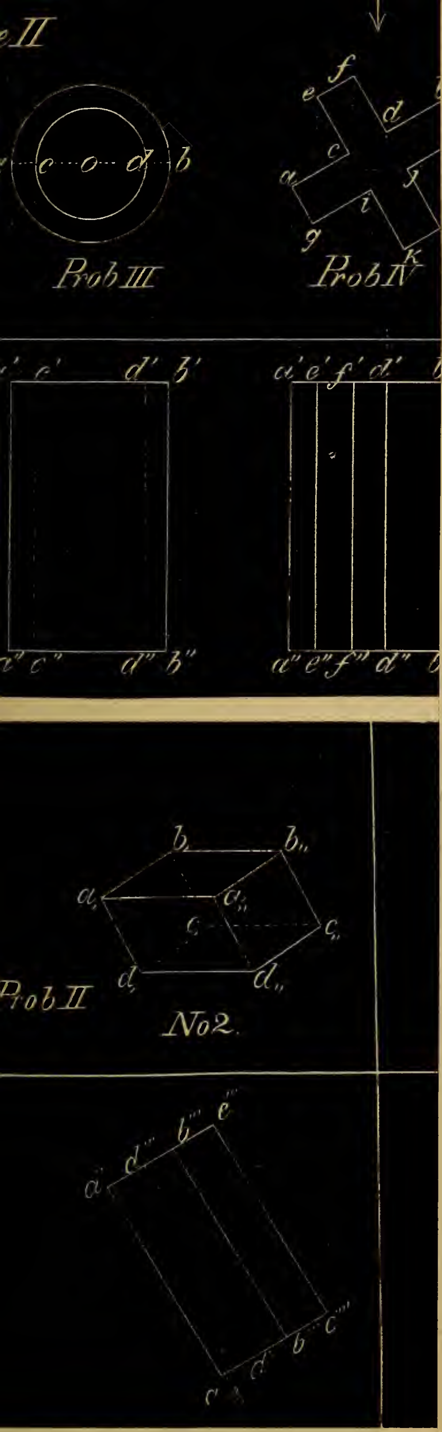


PLATE A.






e II
Prob III
Prob IV
Prob II
No2.

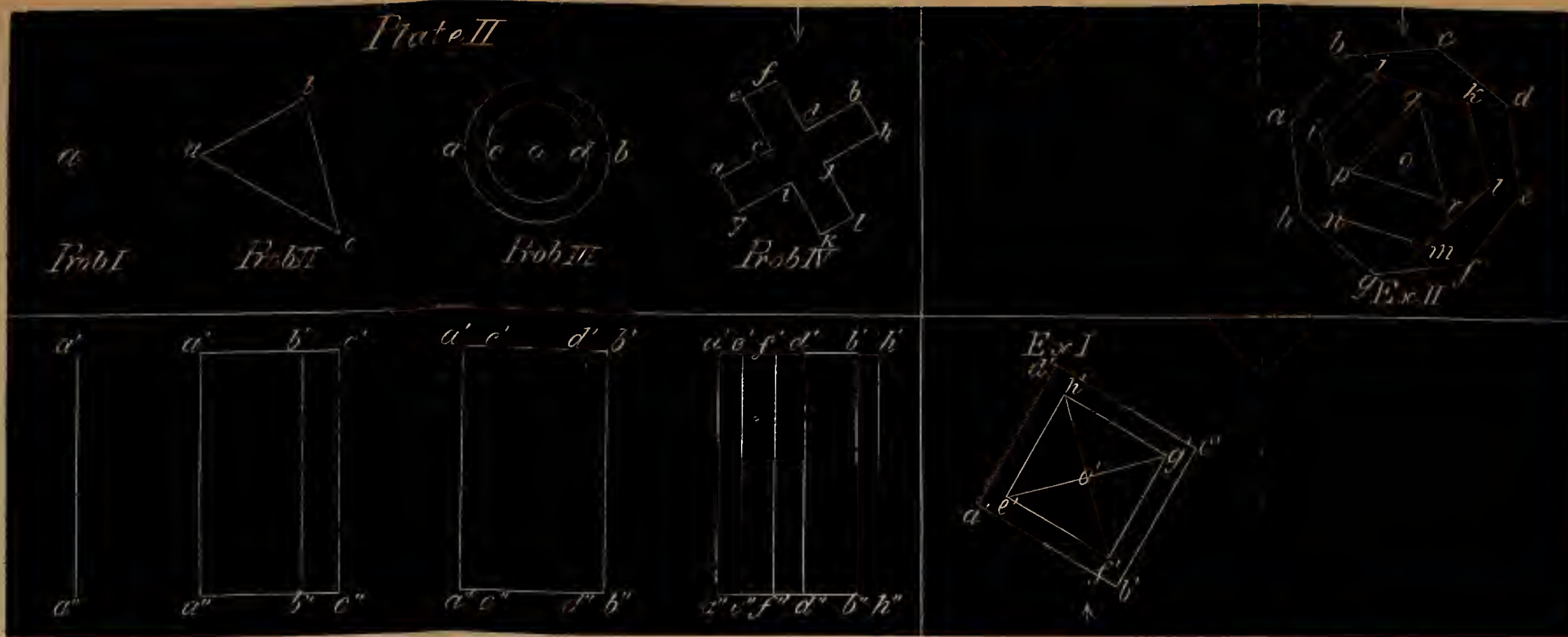


Plate II
Prob I
Prob II
Prob III
Prob IV
Ex I
Ex II

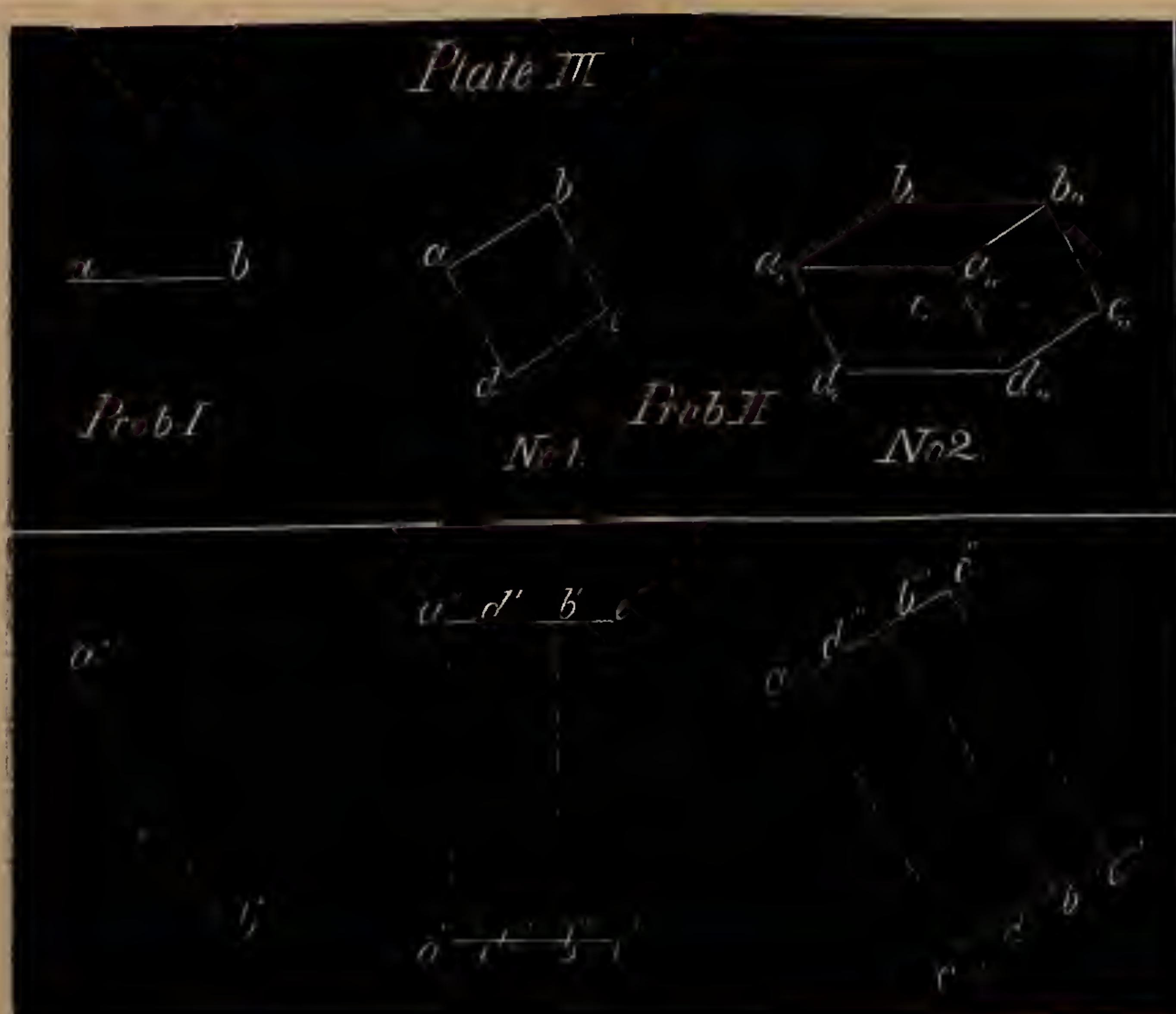


Plate III
Prob I
Prob II
No 1
No 2

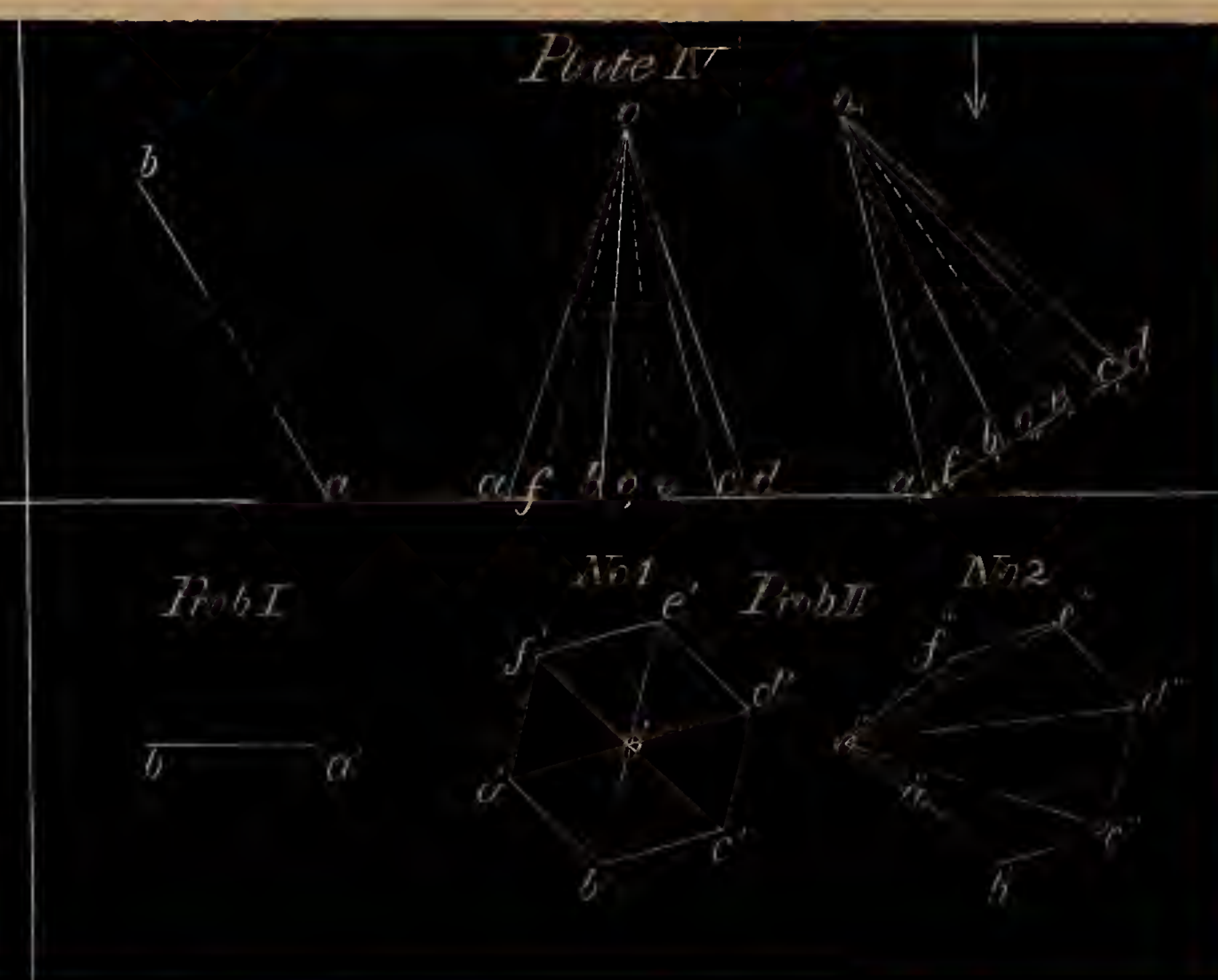
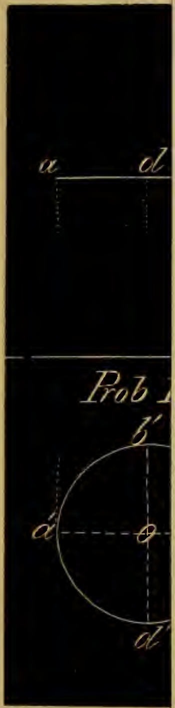


Plate IV
Prob I
Prob II
No 1
No 2


a
a'


a
d'
Prob
b'
o

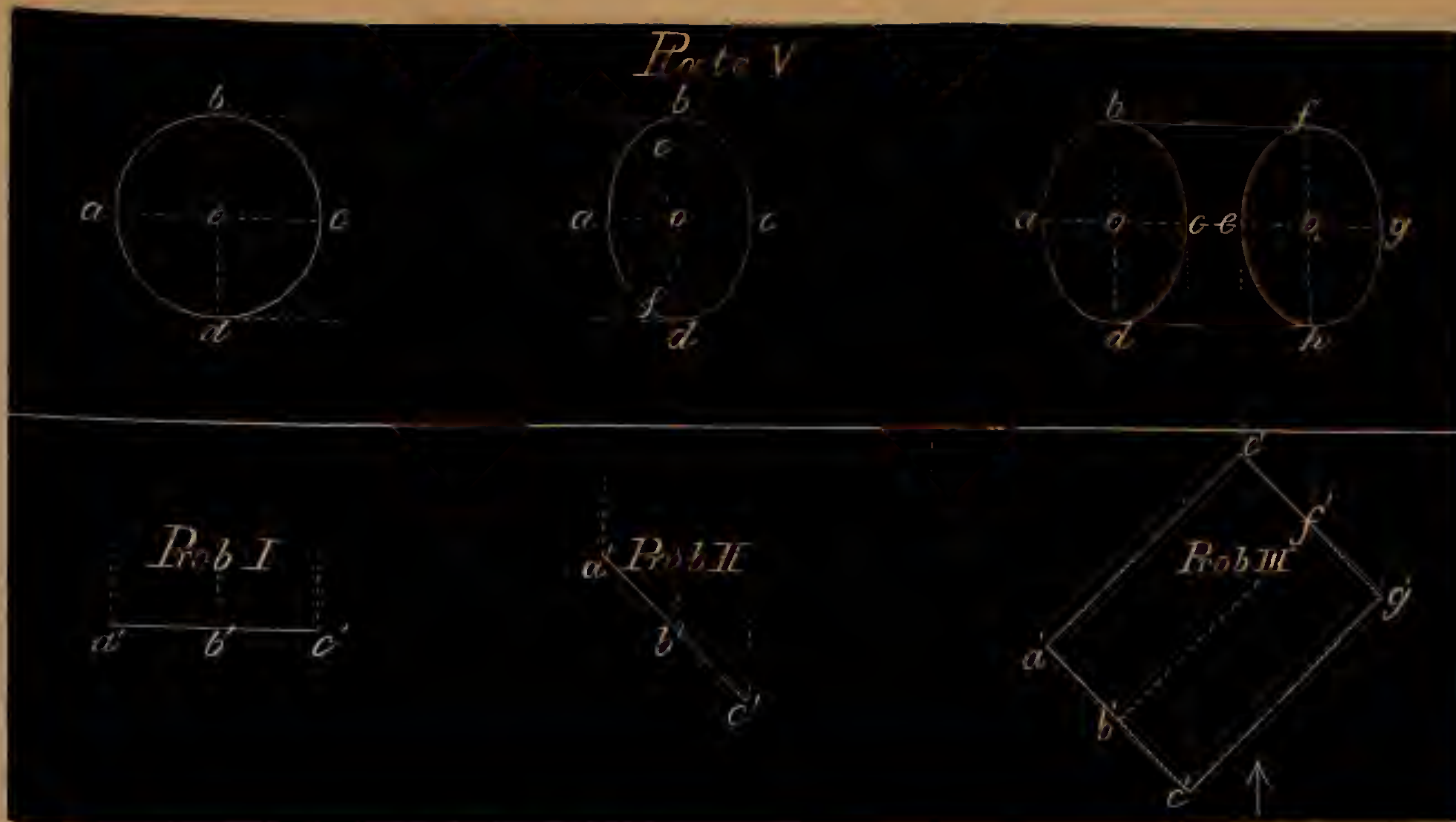


Plate V
Prob I
Prob II
Prob III

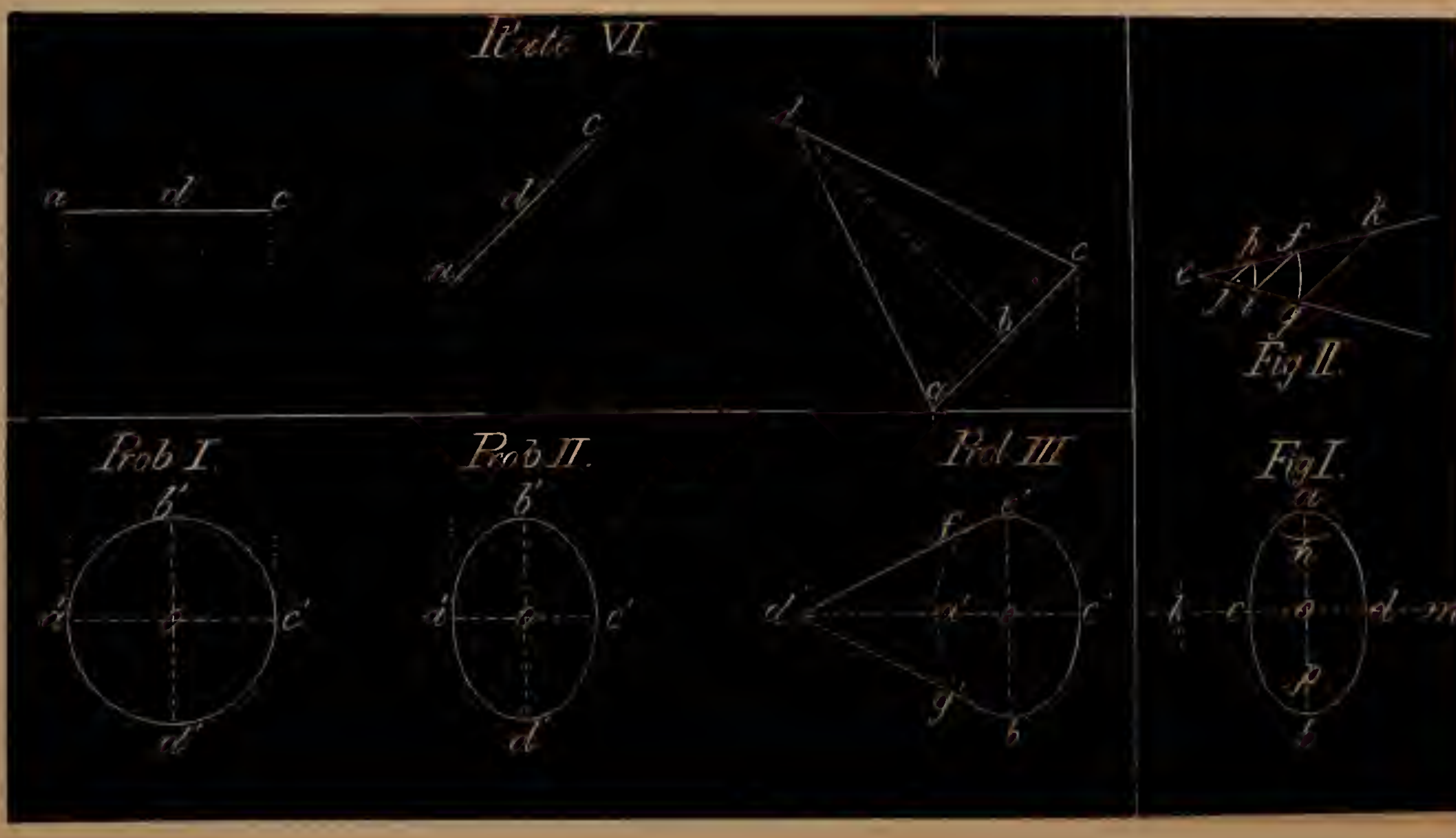
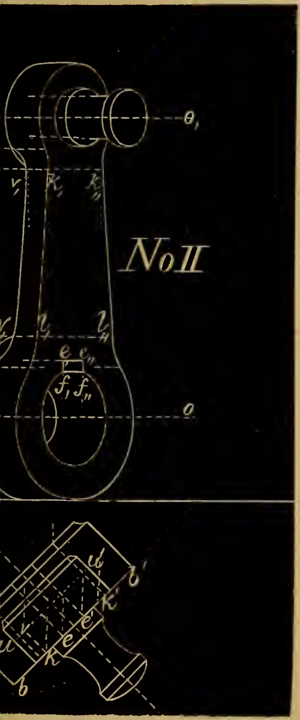


Plate VI
Fig II.
Prob I.
Prob II.
Prob III
Fig I.


d'
a'
e'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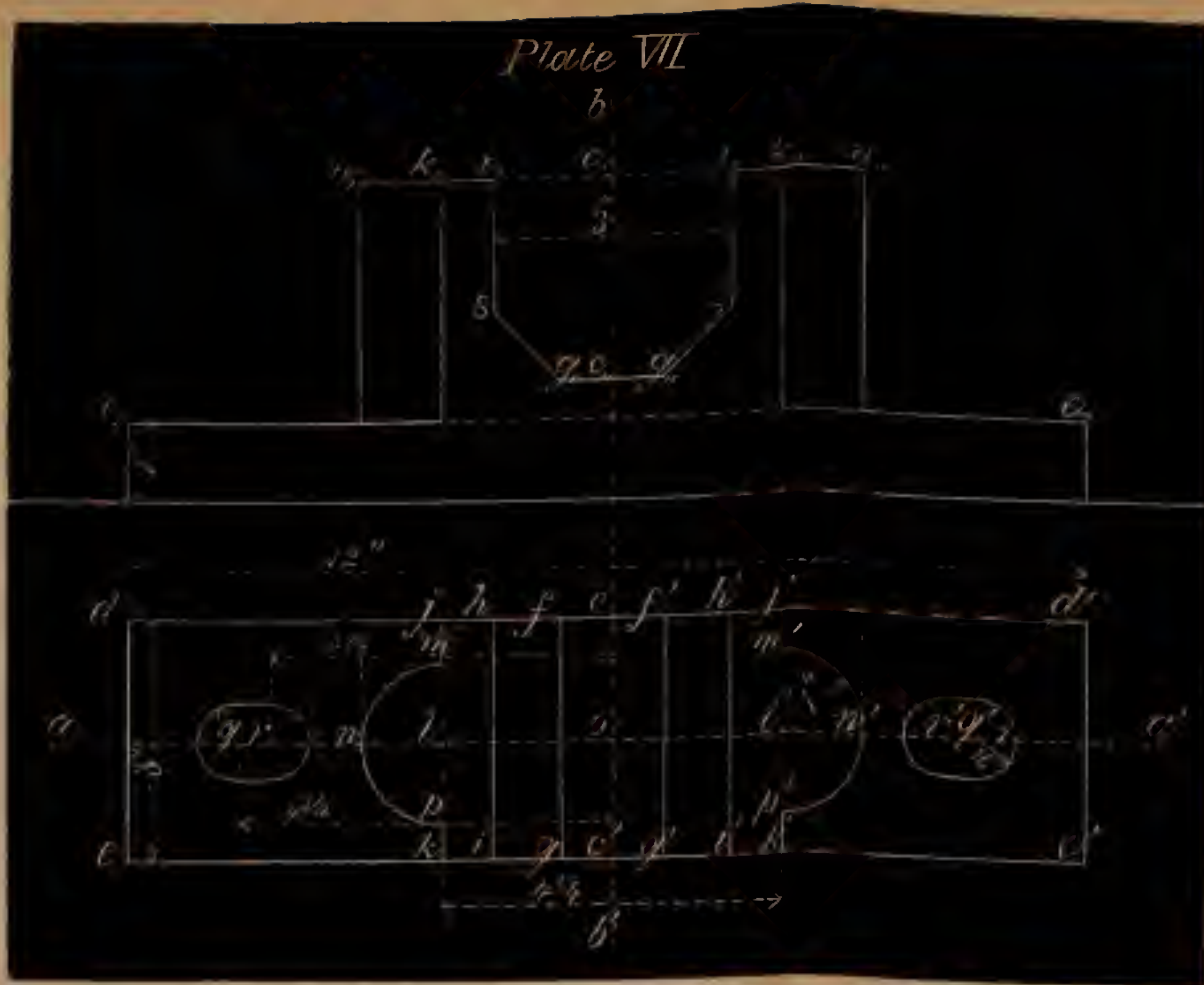


Plate VII

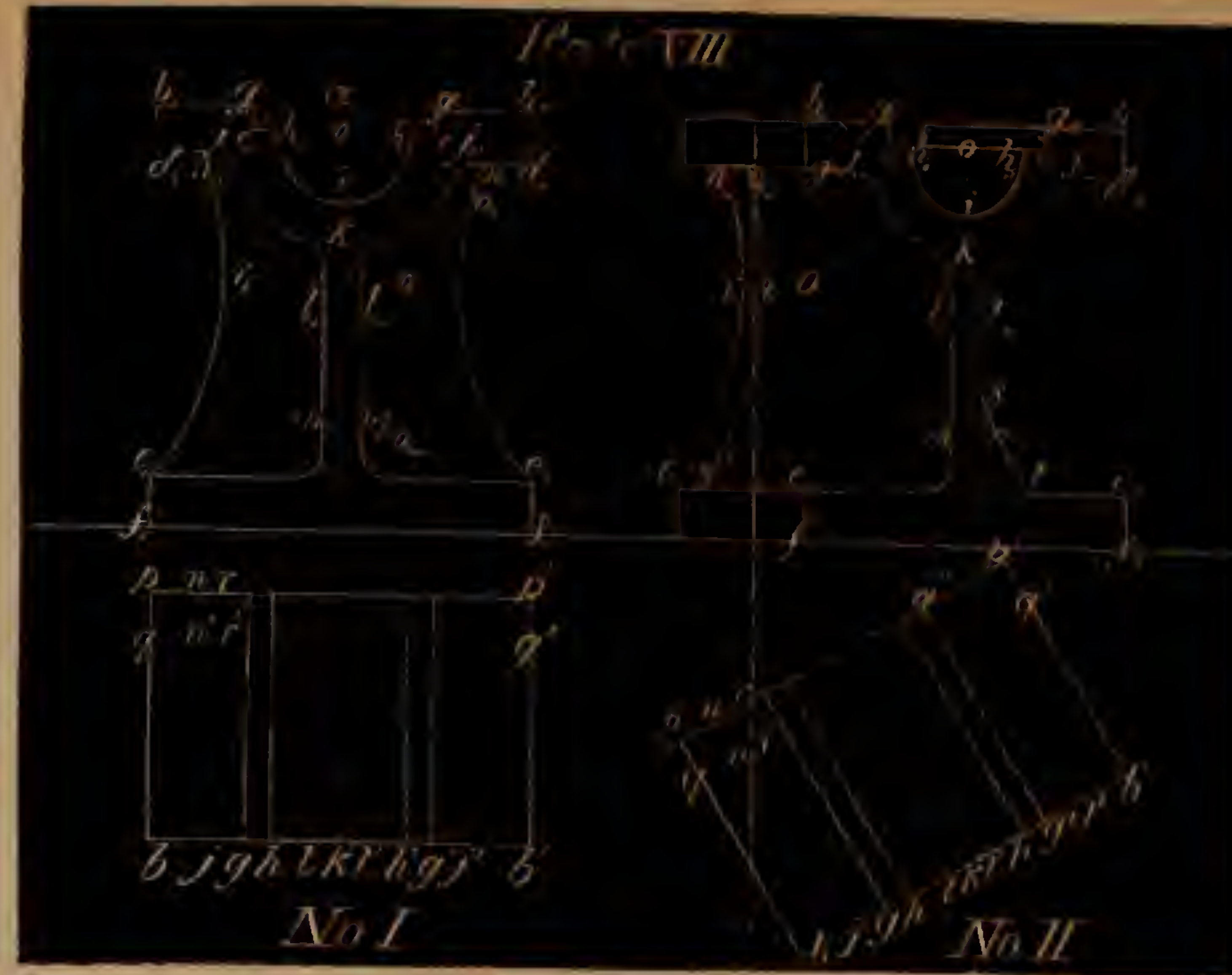


Plate VIII
No I
No II

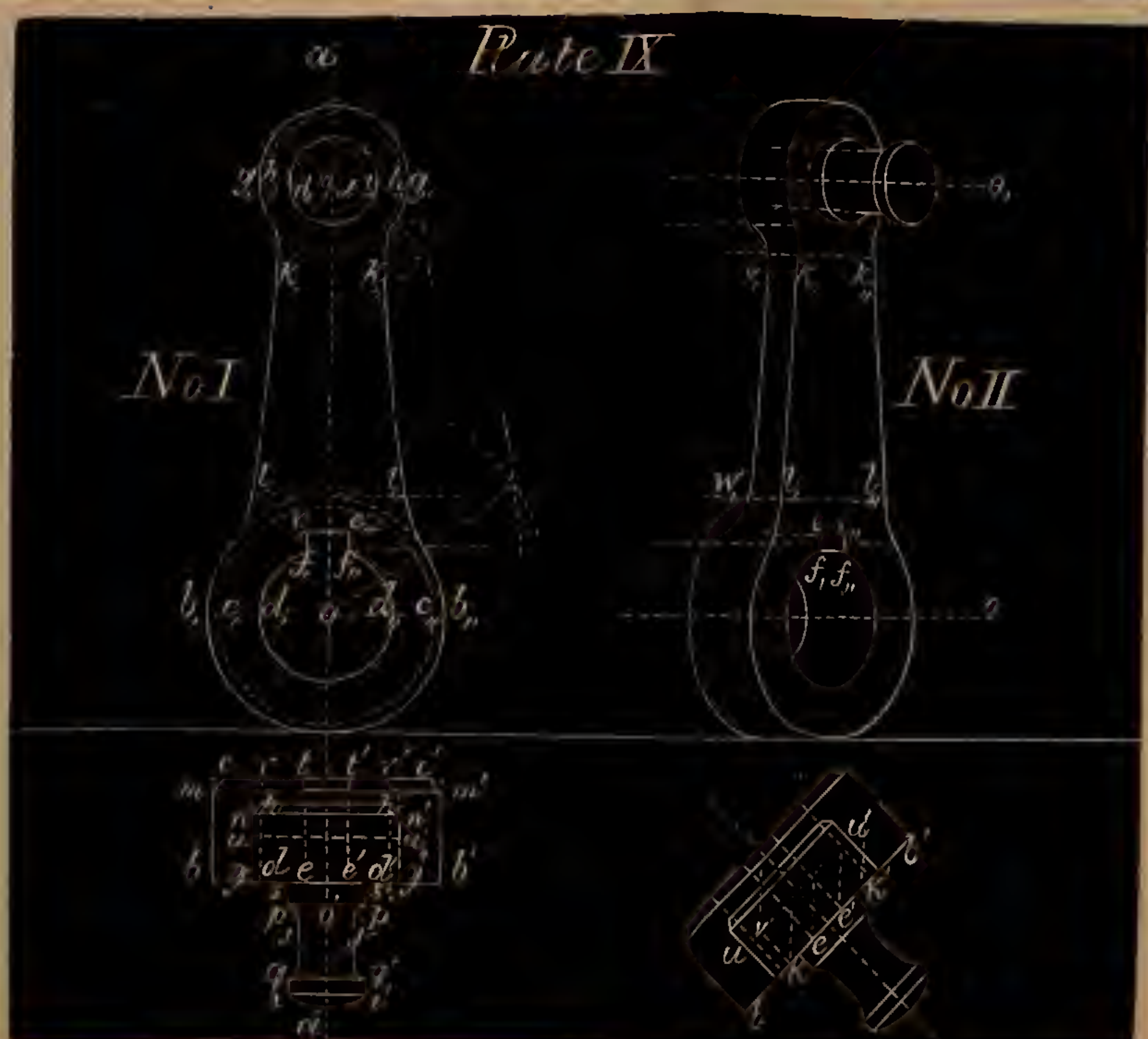
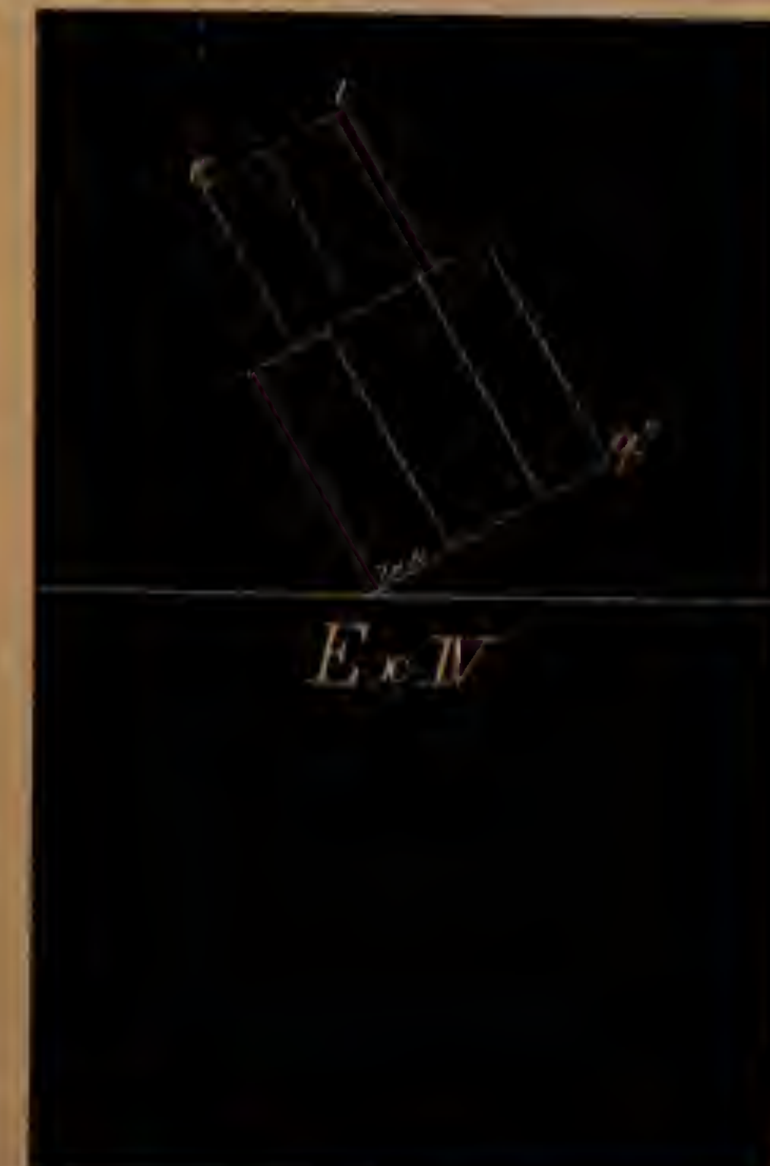


Plate IX
No I
No II


Ex. III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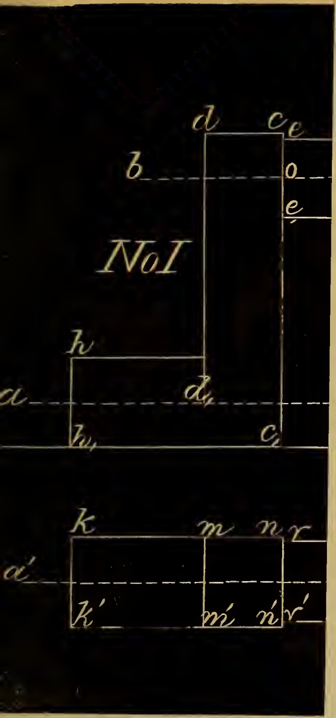
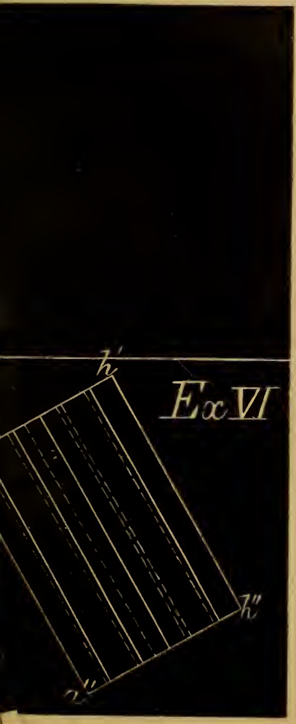

No I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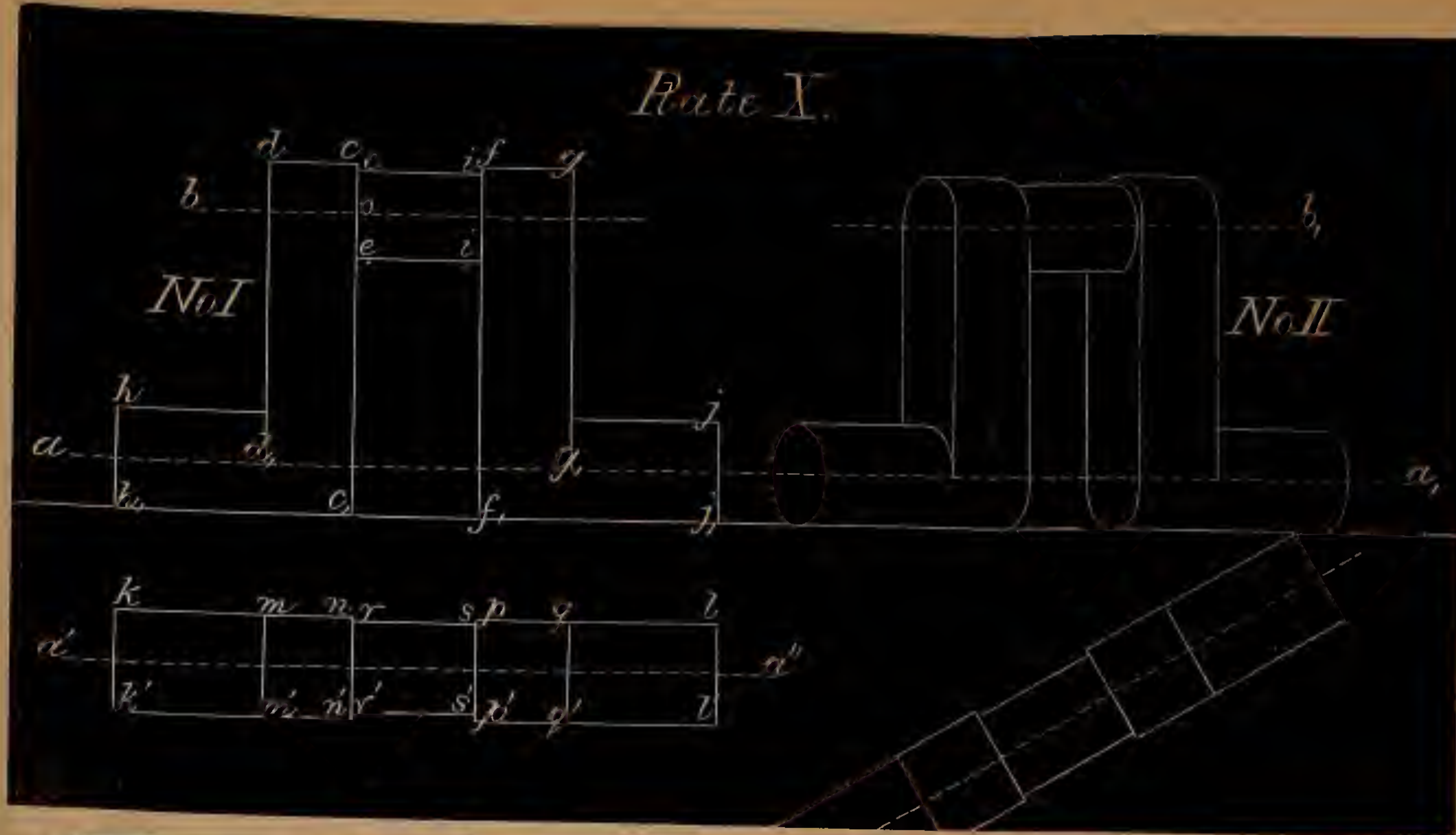
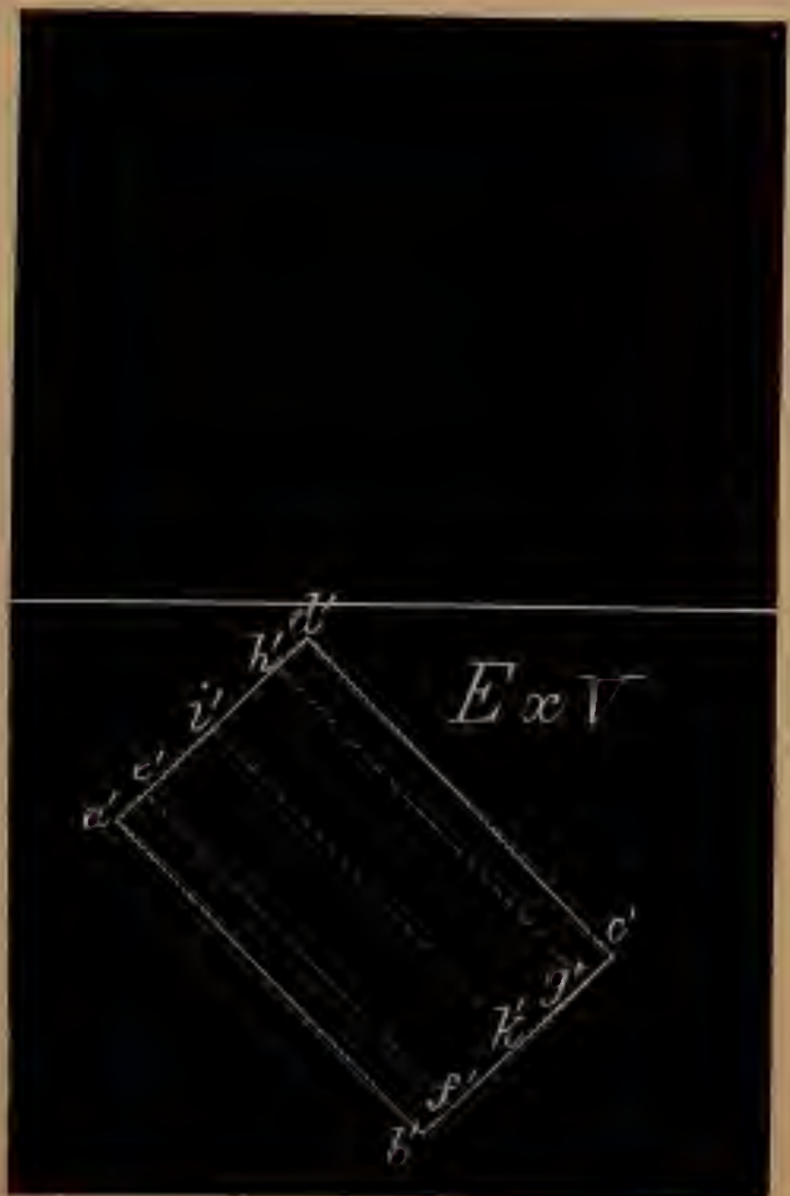


Plate X.
No I
No II


Ex V


Ex VI

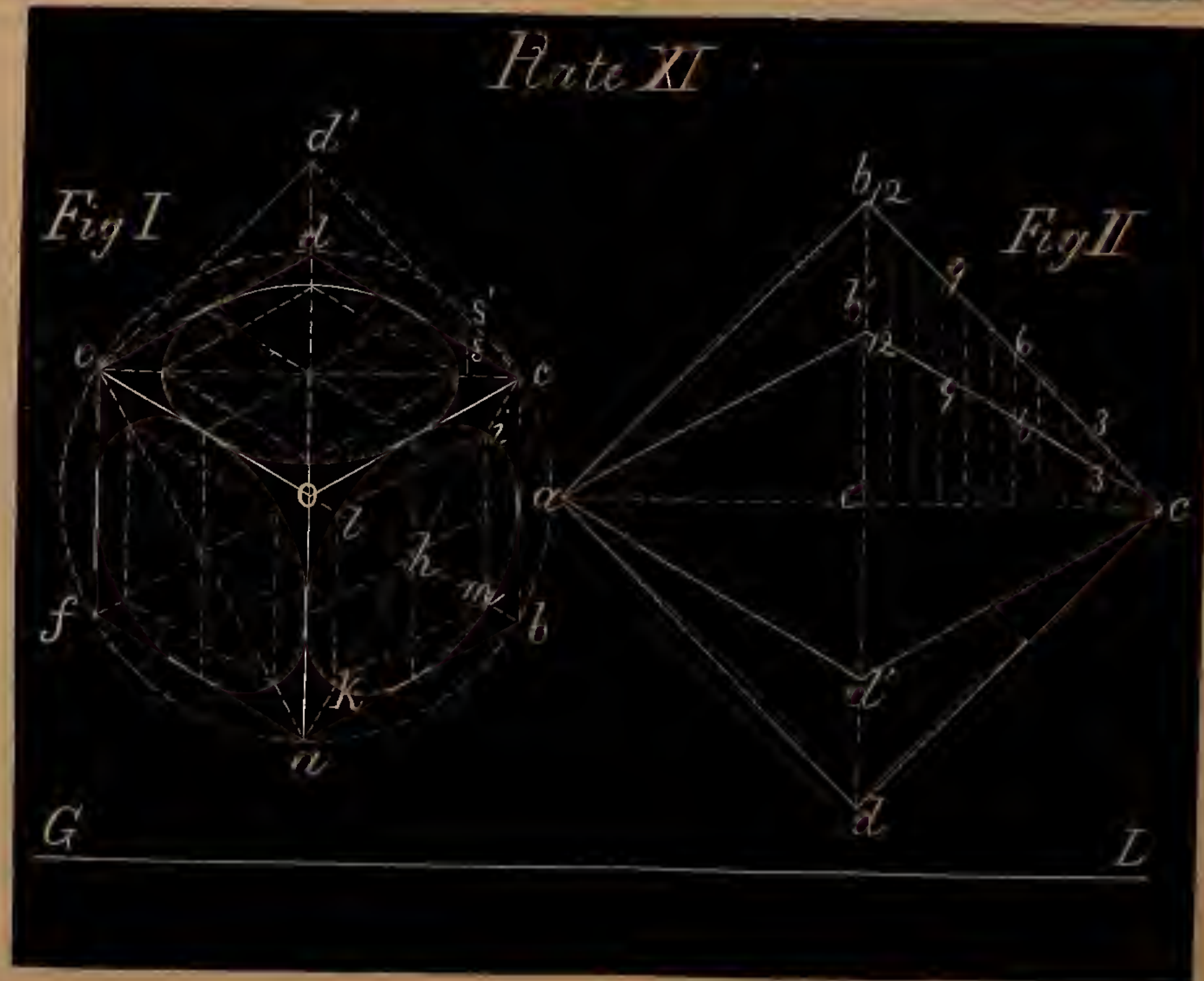
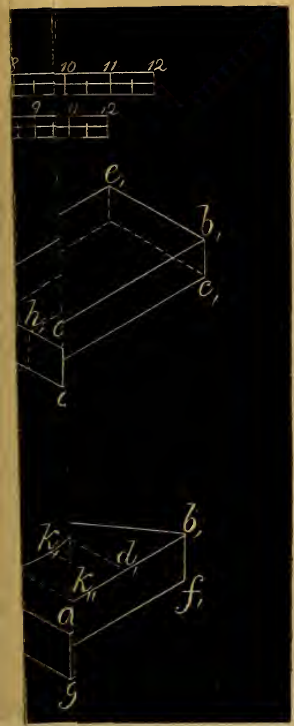


Plate XI
Fig I
Fig II


10 11 12
9 11 12
e,
b,
c,
h,
b,
k,
d,
k,,
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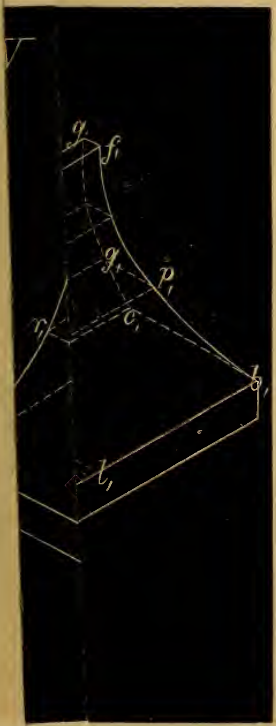
g


V
q
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b,
t,


y

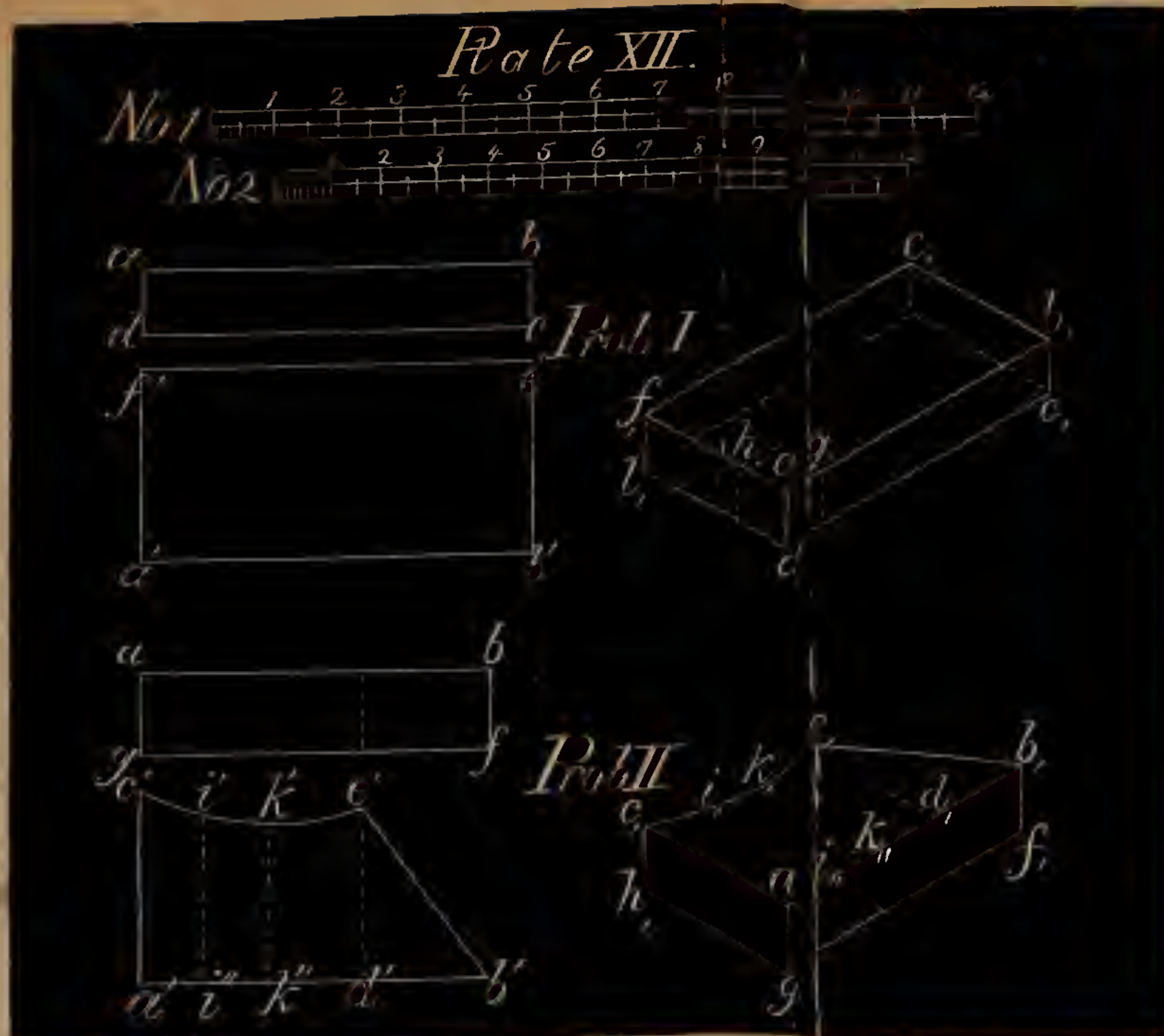
Plate XII.
No 1
No 2
Prob I
Prob II

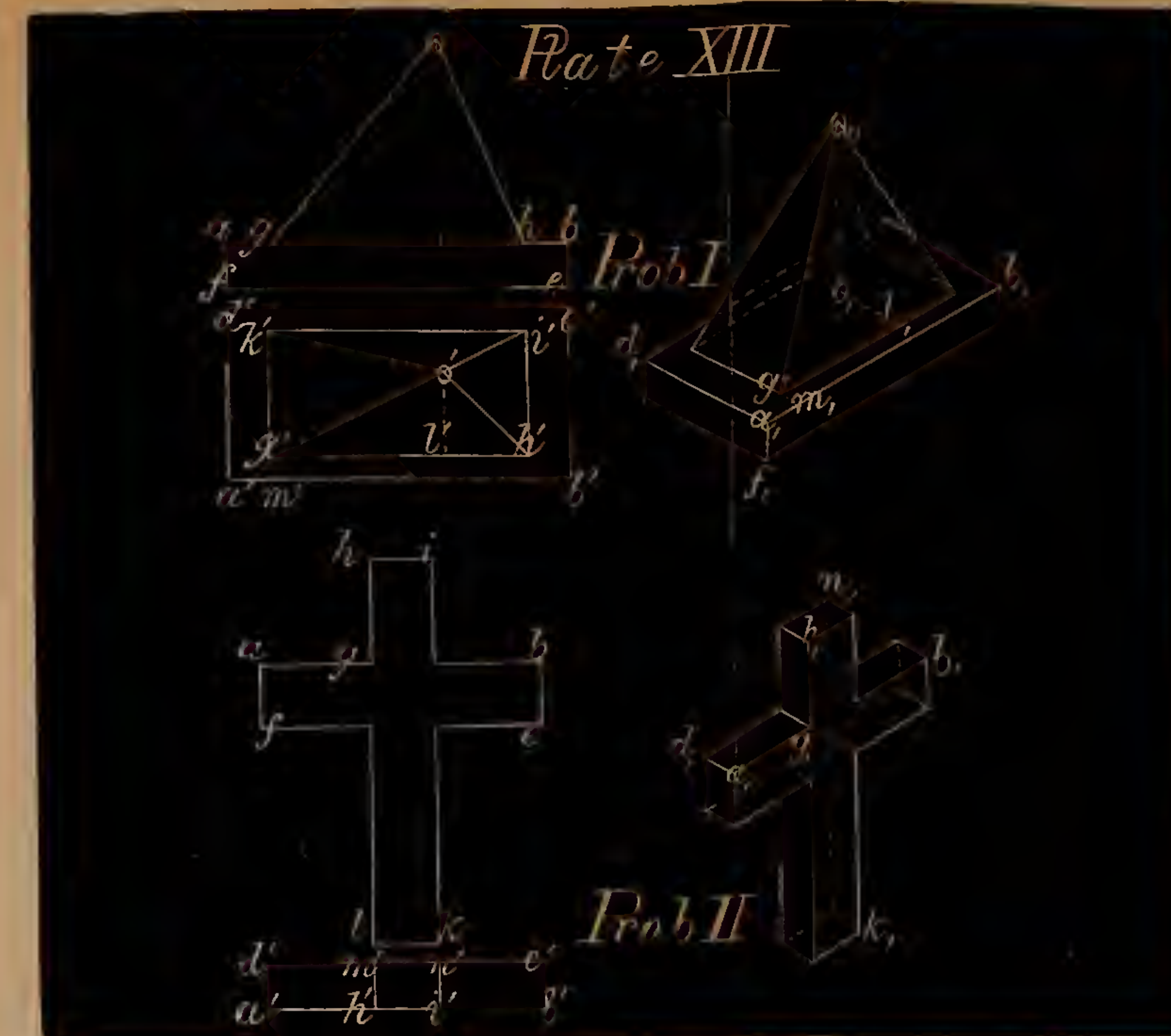
Plate XIII
Prob I
Prob II

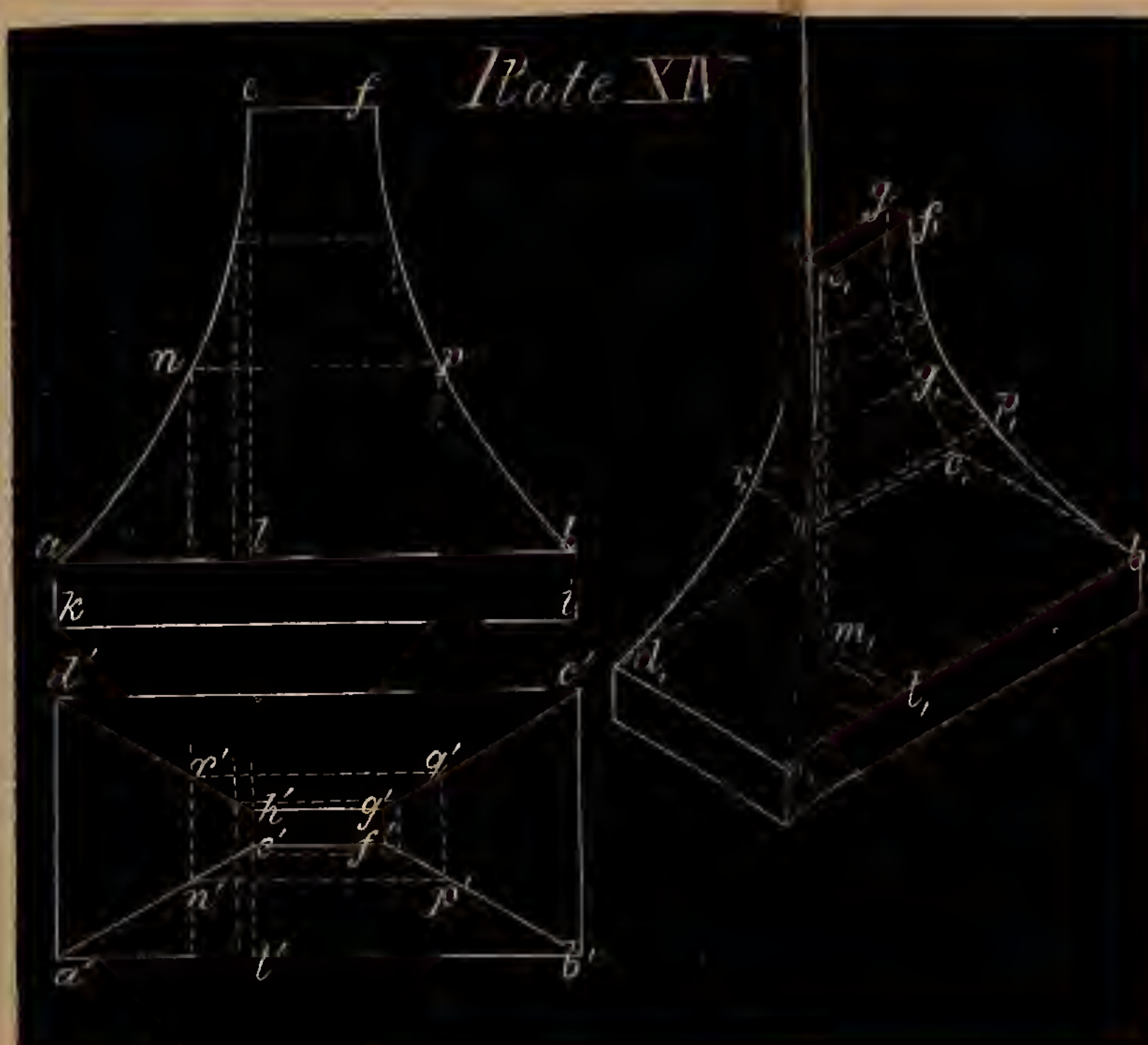
Plate XIV

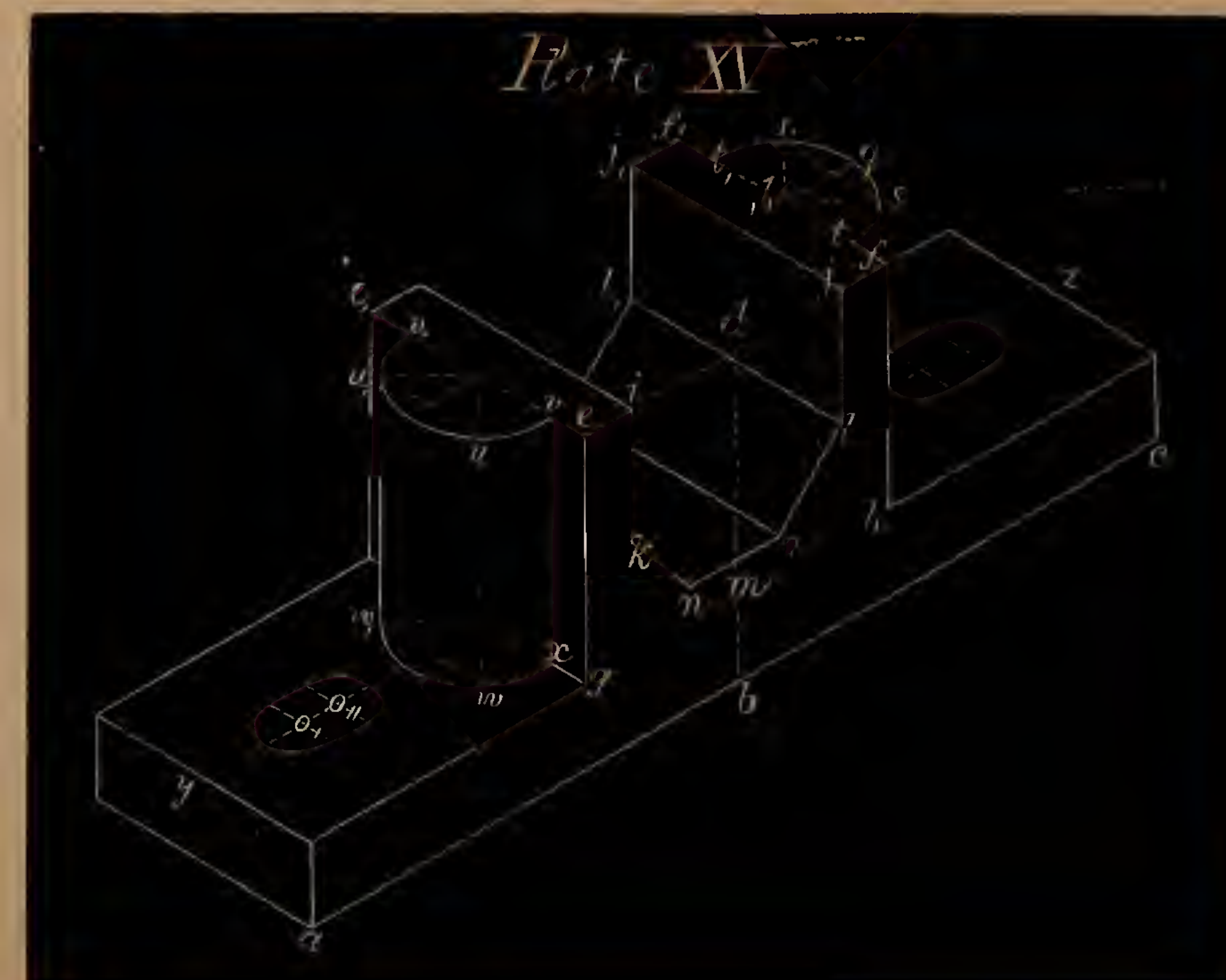
Plate XV